Scale Reading-Brix Conversion Table for the Bausch and Lomb Precision Refractometer in the Normal Weight Solution Method of Analysis of Sugar Products

F. W. WEITZ[1]

The adaptation of the Bausch and Lomb precision sugar refractometer for use in the *Normal Weight Solution Method of Sugar Analysis* described by Bachler (1)[2] for routine determination of Brix of Original Product requires the construction of a special scale reading—brix conversion table, fitted for the instrument scale, supplementing the basic set of tables furnished by the instrument makers for conversion of scale readings to percent sugar in solution.

An element to be recognized in connection with the construction of such tables is the fact that one general table is not universally applicable for all instruments of this type. The table must be adapted to the specific refractive index characteristics of the particular series number prism with which the instrument is provided. The separate lots of prisms cut from glass of different melts are marked for series identification by three digit numbers comprising the last three figures of the index value for sodium light of the particular glass. For example, prisms cut from a lot of glass of a given melt having an index of 1.51716 are identified by the three digit numbers 716. The instrument for which the set of tables herein described was made up is one of a group with prisms of this series, number 716. For the group of instruments of a like numbered prism series one set of tables will be applicable in common, but for each group of a differently numbered prism series a separate set of tables must be made up.

The scale is uniform on all refractometers of the precision type and is graduated in equi-divisions numbered 0 to 72, with the whole numbered divisions subdivided equally into tenths. By means of a vernier the scale setting is read to one tenth of the smallest subdivision, or to one hundredth of a scale unit. The scale reading for the setting on distilled water, $n_D^{20^\circ}$ 1.33299, is in the range of about 21, and the setting for a sugar solution containing 26 grams of dry sugar solids in 100 ml reads on the scale approximately 28. The exact scale division reading values for either distilled water or the normal weight sugar solution depend on the particular prism series.

The table furnished by Bausch and Lomb is identified by like series number with the series number of the prism of the particular instrument,

[1] General Chemist, American Crystal Sugar Company, Denver, Colorado.

[2] The numbers in parentheses refer to literature cited.

Table 1.—Conversion of Scale Readings, B & L Precision Sugar Refractometer, on Normal Weight Sugar Solutions at 20° C into Brix of Solution and Brix of Original Product. —Prism Series 716.

Scale Reading		21.11	21.20	21.30	21.40	21.50	21.60	21.70	21.80	21.90
Brix of Solution		.00	0.35	0.73	1.11	1.49	1.87	2.24	2.62	3.00
Brix Orig. Prod.		.00	1.31	2.79	4.27	5.74	7.21	8.68	10.15	11.62
Scale Reading	22.00	22.10	22.20	22.30	22.40	22.50	22.60	22.70	22.80	22.90
Brix of Solution	3.37	3.74	4.10	4.47	4.84	5.20	5.57	5.93	6.29	6.65
Brix Orig. Prod.	13.09	14.55	16.01	17.47	18.93	20.38	21.84	23.29	24.74	26.19
Scale Reading	23.00	23.10	23.20	23.30	23.40	23.50	23.60	23.70	23.80	23.90
Brix of Solution	7.01	7.36	7.72	8.07	8.42	8.78	9.13	9.47	9.82	10.17
Brix Orig. Prod.	27.64	29.08	30.53	31.97	33.41	34.85	36.28	37.72	39.15	40.58
Scale Reading	24.00	24.10	24.20	24.30	24.40	24.50	24.60	24.70	24.80	24.90
Brix of Solution	10.52	10.86	11.20	11.54	11.87	12.21	12.54	12-88	13.21	13.54
Brix Orig. Prod.	42-01	43.44	44.86	46.29	47.71	49.13	50.55	51.96	53.38	54.79
Scale Reading	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.80	25.90
Brix of Solution	13.87	14.20	14.53	14.85	15.18	15.50	15.82	16.14	16.46	16.78
Brix Orig. Prod.	56.20	57.61	59.02	60.42	61.83	63.23	64.63	66.03	67.42	68.82
Scale Reading	26.00	26.10	26.20	26.30	26.40	26.50	26.60	26.70	26.80	26.90
Brix of Solution	17.10	17.42	17.74	18.05	18.36	18.67	18.98	19.29	19.60	19.90
Brix Orig. Prod.	70.21	71.60	72.99	74.38	75.76	77.15	78.53	79.91	81.29	82.67
Scale Reading	27.00	27.10	27.20	27.30	27.40	27.50	27.60	27.70	27.80	27.90
Brix of Solution	20.21	20.51	20.81	21-12	21.42	21.72	22.02	22.32	22.62	22.91
Brix Orig. Prod.	84.04	85.41	86.78	88.15	89.52	90.89	92.25	93.61	94.97	96.33
Scale Reading	28.00	28.10	28.17							
Brix of Solution	23.20	23.50	23.70							
Brix Orig. Prod.	97.69	99.04	99.99							

or group of instruments, for which it is applicable. The table gives the percent sugar in solution equivalent to scale readings at the temperature of 20° C, by steps of tenths of scale units. Where the solution is made up on the basis of 26.00 grams of sugar product in 100 ml at 20° C. the table values for percent sugar are the values for the Brix of the Normal Weight Solution. These values from the Bausch and Lomb conversion table (2) were transcribed directly as the basis of the Table for Brix of Normal Weight Solution. By simple interpolation between contiguous tenths of scale units the table was expanded to show Brix of Normal Weight Solution by units of one hundredth of scale degrees.

The values for Brix of Normal Weight Solution equivalent to scale readings constitute the basic data for the calculation of the secondary table to show Brix of Original Product. The Table for Brix of Original Product, applicable when the solution contains 26.00 grams of sugar product, weighed in air and made up to 100 ml at 20° C., is constructed of values computed by the following general formula (3):

$$\text{Brix Orig. Product} = \frac{\text{Brix N. Wt. Sol. x Sp. Gr. } 20°/4° \text{ C x } 100}{26.00\ (1 + 0.001\ k)} \qquad (1)$$

to show Brix of Original Product. The Table for Brix of Original Product, applicable when the solution contains 26.00 grams of sugar product, weighed in air and made up to 100 ml at 20° C, is constructed of values computed by the following general formula (3) :

** <** - * - = B - N w ^ - o t ^ 1 2 r ° c , i , < i " <•>

The factor k is a variable, varying with the density of the product, in the reduction of weight of sample in air to weight in vacuo (4) . The density determinant for the appropriate k factor is established from an approximated value for the actual value of the Brix of Original Product. The approximated Brix is derived by applying the known values in the above formula (1) but omitting from the divisor the reduction factor (1 -r 0.001 k) . From the approximated Brix value the density of the product is established by reference to the table of density of sugar solutions at 20° C.

A table for Brix of Original Product, correlated, with the table for Brix of Normal Weight Solution, to scale readings, was compiled for the prism series 716. By the formula as given above (I) , a provisional value for Brix of Original Product equivalent to the scale reading was computed for each one tenth scale unit over the range from distilled water reading to the reading of the normal weight solution of pure sucrose. The procedure of calculation may best be clarified by illustration with an example as follows:

Scale reading	**27.70**
Brix of Normal Weight Solution	**22.32**
Sp. Gr. of solution 20°/4° C. (Sp. Gr. Table)	**1.09134**
Dry Substance in 100 ml, Weight in Vacuo	**24.35871**
Approximate Brix of Original Product.....	**93.69**
Density of product (Plato's Table)	**1.506**
k factor (Table 108, N. B. S. Cir. C440)	**0.657**
Brix of Original Product, provisional	**93.626**

The provisionally established values for Brix of Original Product require minor adjustment for leveling off slight irregularities in the progressive increments for increasing scale readings. The values for percent sugar as given in the Bausch and Lomb conversion table, which were translated to the prepared table as the basic values of Brix of Normal Weight Solution, are the sucrose equivalents of the index of refraction corresponding to the scale readings. These values for percent sugar are interpolated from the 1936 International Table of Refractive Indices for Sugar Solutions at 20° C and are rounded to the second decimal. The magnitude of deviation on rounding may be of the order of ± .005 Brix. As the ratio of the Brix of the Normal Weight Solution to the Brix of Original Product is approximately 1:4, the calculated value provisionally established for the Brix of Original Product may show deviation from the most probable value by as much as 4 times ± 0.005 or ± 0.02 Brix. Other slight deviations in the table from what would appear to be a normal smooth curve in the relationship of scale reading to percent sugar may offset the calculated

Brix of Original Product value by as much as ± 0.04 in some instances. The effect of these deviations is a table of computed values of Brix of Original Product showing slight irregularities from a smooth curve of progressive increments of Brix for uniform stepwise increments of scale units.

To correct for these apparent discrepancies the provisional values established by calculation from the general formula for Brix of Original Product were adjusted to a smooth curve by recalculation from the normal equation to the line of best fit derived by the method of least squares. Plotting the

Table 2.—Temperature Corrections for Scale Readings on Bausch and Lomb Precision Sugar Refractometer, Standard at 20° C.—Prism Series 716.

Temp. °C	Scale Reading 21.11	22	23	24	25	26	27	28	29	Temp. °C
					Subtract from the Observed Scale Reading					
18.0	0.031	0.034	0.036	0.038	0.041	0.043	0.046	0.048	0.051	18.0
.5	.023	.025	.027	.028	.031	.032	.034	.036	.038	.5
19.0	.016	.017	.018	.019	.021	.022	.023	.024	.026	19.0
.5	.008	.008	.009	.010	.010	.011	.012	.012	.013	.5
					Add to the Observed Scale Reading					
20.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	20.0
.5	.008	.009	.009	.010	.011	.011	.012	.013	.013	.5
21.0	.017	.018	.019	.020	.021	.023	.024	.025	.026	21.0
.5	.025	.027	.028	.030	.032	.034	.036	.038	.039	.5
22.0	.033	.036	.038	.040	.043	.045	.048	.050	.053	22.0
.5	.042	.045	.048	.051	.054	.057	.060	.063	.066	.5
23.0	.051	.054	.058	.061	.065	.068	.072	.076	.079	23.0
.5	.060	.064	.068	.072	.076	.080	.085	.089	.093	.5
24.0	.069	.073	.078	.083	.088	.092	.097	.102	.107	24.0
.5	.078	.083	.088	.094	.099	.104	.110	.116	.121	.5
25.0	.088	.093	.099	.105	.111	.117	.123	.129	.135	25.0

Brix values against scale reading shows the relationship to be described by a slightly curved line of the parabola type. The most probable values fitted to the curve were computed from the formula of a second degree equation to a curve of the parabola type. In its generalized form this is expressed (5) as

$$y = a + bx + ex^2$$

where, for the curve in this case,

x = Scale reading in increments of 0.1, and

y = Provisional values of Brix of Original Product as computed by formula (I) for scale readings x.

Following common procedures of statistical analysis in computing trend lines, the values of a, b and c for substituting in the formula may be derived by resolving simultaneously from the relationship expressed in the equations

$$5\ (y) \ - na \qquad + b\ 2\ (x) \ + c\ 2\ (x^2)$$
$$2\ (xy) \ = a\ S\ (x) \ + b\ \wedge\ (x^2) \ + c\ \wedge\ (x^3) \qquad \text{(III)}$$
$$2\ (x^2y) \ = a\ 2\ (x^2) \ + b\ 2\ (x^3) \ + c\ 2\ (x^*)$$

n— number of items of paired data. In this case n = 71 = the number of correlated items of scale readings x and the Brix equivalents y, in 0.1 scale divisions. The scale reading divisions in 0.1 units were taken from the first whole 0.1 scale unit above the reading for water to and including the next whole 0.1 scale unit above the reading corresponding to the Brix of the Normal Weight sugar solution.

Taking advantage of a short cut procedure in the statistical analysis trend line computations, applicable where the values of x are consecutive and n is an odd number, the labor of calculations can be materially curtailed if, instead of describing the curve with origin at the scale reading for zero Brix, the origin of the curve is shifted to the median value of the scale range. In this case, n = 71, the median value is 36 in the order of sequence of consecutive items. The median scale reading value x may be represented by o and the other values of x in progression each way from the median value represented by the consecutive numbers 1 to 35, respectively, minus and plus. In that representation 2 $(^x)$ and 2 (x^3) become zero. The above simultaneous equations (III) then are reduced to the following simpler forms;

$$2\ (y) \ = na \qquad + c\ 2\ (x^2)$$
$$2\ (xy) \ = b\ 2\ (x^2) \qquad \text{(IV)}$$
$$2\ (x^2y) \ = a\ 2\ (x^2) \ + c\ 2\ (x^4)$$

Inserting the known values of the sums of the x's and y's of the indicated powers and products, the equations are solved for a, b and c. Substituting these values then in the general equation of second degree to a curve of the parabola type (II), the normal equation derived for the line of best fit for values of y, Brix of Original Product, correlated to scale reading values for x, for prism series 716, was

$$y = 51.9616 + 1.41548\ x — 0.0009022\ x^2 \qquad \text{(V)}$$

From this equation (V) the respective values of y, Brix of Original Product, for each 0.1 scale reading value x were then recalculated. The values of x representing 0;1 scale readings for inserting in the equation were the respective substituted consecutive series numbers 1 to 35, plus and minus, each way from the median value taken as 0," Intermediate values of y by steps of 0.01 scale reading for the expanded table were derived by interpolation between the computed values for contiguous 0.1 scafe units. All final calculated values were rounded to the second decimal. In abridged form, for units of 0.1 scale readings, the Brix conversion table, prism series 716, is reproduced herein as Table 1.

Based on the data of Schonrock for temperature corrections on refractive index of sucrose solutions (6), a Table of Temperature Corrections on scale readings for the Bausch and Lomb refractometer was prepared, shown as Table 2- The table gives the corrections to be applied directly on observed scale readings in the temperature range of 18° to 25° C. for adjustment to the equivalent readings at the standard temperature of 20° C.

Literature Cited

(1) BACHLER, F. R.
1933. One Solution Method of Analysis. Facts About Sugar, 28, p. 240.

(2) BAUSCH AND LOMB OPTICAL COMPANY
Precision Refractometer Conversion Tables, Series 716.

(3) LANDT, E.
1933. Ueber ein neues Eintauchrefraktometer von Zeiss. Zts. ver d. Z. Ind. 83, P. 692-716.

(4) ————————
1942. Polarimetry, Saccharimetry and the Sugars. N.B.S. Cir. C440, Table 108, p. 613.

(5) MILLS, FREDERICK CECIL.
1938. Statistical Methods, Henry Holt & Co., p. 253-256.

(6) SCHONROCK, O.
Temperature Corrections for Determining Refractive Index of Sucrose Solutions. N.B.S. Cir. C440, Table 127, p. 664.