The Development of Control Charts for Package Weights

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Received for publication February 28, 1962

Weight control on a package line has always been subject to question. The question of how often should samples be taken, when is a scale adjustment required, what constitutes a light weight, how much overweight is required, how to tabulate and evaluate check weighings and many more come up whenever the subject of weights arises. Most of the questions can be answered through the construction and use of control charts.

Control chart technique has proven to be a valuable tool as evidenced by rapidly expanding use in industry and governmental agencies over the past several years. The literature contains a great deal of information on this subject and on related statistical problems. A list of what we believe to be excellent references will appear at the end of this paper.

Each weighing system presents a somewhat different problem. Also each piece of equipment yields a weight distribution pattern around some mean or average weight produced. This paper will outline and briefly discuss the steps necessary to set up Weight Control Charts for multiple scale equipment used in the production of five- and ten-pound packages of granulated sugar. The same general procedure can be applied to other size packages.

Unfortunately, it is a rare occasion when the same control chart can be applied to two pieces of equipment apparently identical in every respect. Furthermore, from time to time the performance of the equipment changes due to mechanical wear, difference in product and other causes. Consequently, it is necessary to construct a Control Chart for each multiple head unit and to re-evaluate the performance of any unit from time to time or after a major overhaul.

A Control Chart program can be developed and put into operation by following the steps presented herein. For more detail and information relative to the derivation of the mathematical relationships the reader is referred to the literature list on this subject.

Calibration of Check Scales

If the scales used for check-weighing show only over or under and exact weight, it is necessary to inscribe calibration marks on the scale dial. Our company uses Toledo check scales for the five- and ten-pound packages. These scales can be calibrated in five-gram units and can be read to two and one-half grams. The

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sensitivity of the scales is such that a one-gram weight will deflect the pointer slightly with a ten-pound load on each side of the scale. The five-gram division has been found to be more satisfactory than a one-quarter ounce division. It is necessary to select a division small enough to yield a satisfactory distribution curve for the package being produced from the equipment. The calibration and subsequent use of the scale divisions are easier to handle if they are referred to as units rather than their actual value. In this case one unit is equal to five grams but the data are recorded in units and half units and not as grams.

Determination of Tare Weight

In practice the sewn bag is used for check-weighing rather than the unsewn bag. This procedure eliminates the chance of spilling sugar or otherwise spoiling the sample. It also makes it easier to sample the production line.

During the sewing operation a small amount of the bag top is clipped off and the tape plus thread is added. The net change in the weight of the empty bag caused by this step should be determined. It usually will not exceed 1.2 grams.

A random sample of 15 to 20 bags must be withdrawn from each of several lots of empty bags to determine the average weight of the empty container. A tare weight equal to this average minus the sewing loss is then made from an old weight or some other suitable material. The weight for the empty bag should be determined for each shipment of empty bags. This weight is used with the appropriate five- or ten-pound weight used on the check scales.

The practice of using an empty bag for a tare weight is satisfactory only if it is adjusted for clip-off and is changed each day or so. Paper is subject to weight change caused from moisture changes and dust.

The check scales and weights must be kept clean, free from vibration and checked for zero balance at all times.

Evaluation of Packaging Equipment Variability

In most instances, packaging equipment for five- and ten-pound units consists of four or six scale buckets, filled and emptied in sequence. Variability in delivered weights is subject to the total of several effects. Sugar condition, cleanliness of the linkages, vibration, sensitivity of mercoid switches or sensing devices, mechanical sequencing and gearing, all have an effect on the uniformity of the delivered weight. To estimate the variation it is necessary to collect and check-weigh thirty to fifty sets of packages produced over a period of three or four hours. A set consists of six bags (one from each bucket in sequence) for a 6-bucket scale unit. The results of the first five sets are examined first to determine whether or not the six buckets are all the same in delivered weights. If any are found to be consistently underweight or overweight, that bucket is adjusted to be more nearly equal to the correct amount or in line with the others. Sampling is then continued, without further scale adjustment until the thirty or more sets are obtained and the results tabulated as shown in Table 1.

Table 1.—Typical package machine data for control chart development. Nampa Factory 5 lb. Machine—6 Buckets, 1 Unit = 5 Grams

Test no.	1	2	3	4	5	6		Avg.(1)	R(1)
1	1.0	—.5	0	1.5	3.5	2.5	8.0	1.3	4.0
2	0.5	1.0	0.5	2.0	2.5	1.5	8.0	1.3	2.0
3	1.5	2.5	1.5	1.0	—1.5	1.5	6.5	1.1	4.0
4	2.0	4.0	2.0	3.5	3.5	2.5	17.5	2.9	2.0
5	0.5	—1.0	1.0	1.0	0	2.0	1.5	0.3	3.0
6	1.5	0.5	1.5	3.5	1.5	—1.0	7.5	1.3	4.5
–	—	—	—	—	—	—	—	—	—
Through	—	—	—	—	—	—	—	—	—
–	—	—	—	—	—	—	—	—	—
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31	2.0	—0.5	0.5	0	—2.5	1.5	1.0	0.2	4.5
Total 31	43.5	40.0	33.0	51.0	45.5	43.0	256.0		93.1
Avg(2)	1.4	1.3	1.1	1.6	1.5	1.4			
R (2)	4.0	6.0	3.5	5.0	6.0	7.0		$\overline{R}$ (1) =	3.00

$$\overline{R}(2) = \frac{31.5}{6} = 5.25$$

$$\overline{R}(1) = \frac{93.1}{31} = 3.00$$

$$\sigma' = \frac{R}{d_2} = \frac{3.00}{2.534} = 1.18$$

Results are recorded as units and half units with a negative sign indicating the lightweight packages. An algebraic sum, average and range is calculated for each set and for each individual bucket. The range is defined as the total difference (in units) between the lightest and heaviest item in the set or series.

At this point there is some difference of opinion as to the proper evaluation of the results. Strictly speaking the correct way to evaluate the results would be to consider each bucket separately since each bucket can be individually adjusted. However, this would necessitate a separate control chart for each bucket. Sampling and plotting results would be complicated and too time consuming for efficient control. The danger of mathematical errors or lack of full understanding on the part of employees could more than offset the slight difference between this approach and the simplified method outlined as follows.

In actual practice the four or six buckets empty in sequence for each cycle. If an inspection by Federal or State agencies is made at a retail outlet for sugar, all of the sugar in that lot can usually be assumed to have come off the production line during some continuous period from a few minutes to a few hours. If twenty to twenty-five packages are examined they may well represent only three to four cycles very close together. It is preferable in our opinion to consider the calculation of the scale unit accuracy as sets consisting of four or six packages, one from each scale bucket in sequence.

An examination of Table 1 will usually reveal a wider range within buckets than between the separate buckets. This fact has the effect of tightening the control slightly which will be more evident when the Control Chart Development is completed.

Since we are considering the accuracy of the equipment as an entire unit and not as individual components, it is next necessary to calculate the sum of the set ranges and an average range for the number of sets involved. The standard deviation (σ') of the set ranges is either calculated or taken from Statistical Tables for Range. Table 2 reproduces in part the necessary factors involved. By using the values in Table 2

$$(1) \quad \sigma' = \frac{\overline{R}}{d_2}$$

σ' is, therefore, an expression for the scale accuracy or the measure of the dispersion about the Range Mean.

Selection of the Amount of Average Overweight

Unfortunately, it is not permissible for a producer of a packaged commodity to market a product which averages the stated net weight stamped on the bag and be in agreement with the various State regulations. If this were so, 50% of the packages would conceivably be slightly overweight packages and 50% slightly underweight. In a broad sense the average accepted weight must not be less than the stated net weight and only a reasonable amount or percent of packages can be underweight. The actual amount of underweight expressed as a percent of the net on those underweight packages must not be excessive. The definition of "excessive" is rather vague at this point.

In addition, the average weight from any production line rises and falls over a period of time. This is caused by machine inaccuracies or product fluctuations or both.

The decision as to the percent underweight a company will decide to produce determines the average over-fill sacrificed in order to meet the specifications. The precision of the packaging equipment enters into the discussion at this point. Equipment which produces reliable weights within narrow limits or dispersion permits the company to bring the average weight closer to the net weight than less reliable equipment yielding a wide variation in weights of product.

Generally speaking, the underweight percentage will be between 10 and 35%. As a rule of thumb, it is generally permissible to increase the percent underweight as the package net weight is increased. This reasoning can be used to standardize the percent over-fill on total product for all size packages a company is willing to accept providing the equipment permits the producer to comply with regulations.

Table 2.—Condensed table of factors for establishing control chart limits.

No. of items in Set	Factors			Percent underweight desired	Factor Z
	d_2	A_2	D_4		
2	1.128	1.880	3.267	0	3.00
3	1.693	1.023	2.575	2.5	1.96
4	2.059	0.729	2.282	5.0	1.64
5	2.326	0.577	2.115	7.5	1.44
6	2.534	0.483	2.004	10.0	1.28
				12.5	1.15
				15.0	1.04
				17.5	.93
				20.0	0.84
				25.0	0.67
				30.0	0.52
				35.0	0.39

$\sigma' = \dfrac{\overline{R}}{d_2}$

$\overline{X} = \sigma' / z$

3 σ Limits for Average

$UCL = \overline{X} + \overline{R}\,A_2$

$LCL = \overline{X} - \overline{R}\,A_2$

3 σ Limit for Range

$UCL = \overline{R}\,D_4$

Note: Values for σ' $\overline{X}$ and $\overline{R}$ apply to system where 0 = stated Net Weight on Package and values of statistics are in appropriate units as used with check scales.

Source: Complete Tables for above values are found in ASTM Manual on Quality Control of Materials, 1951, and in Probability Tables.

For example let us assume that it is decided to produce 12½% underweight packages as a reasonable amount. In Table 2 under the column Z the value of 1.15 relates 12½% of the one tail area under the normal curve to the standard deviation of the machine accuracy calculated from Table 1, according to the formula:

$$(2) \quad Z = \frac{\overline{X}}{\sigma'} \text{ or } \overline{X} = Z\sigma'$$

where $\overline{X}$ establishes the average overweight necessary to produce 12½% lightweight packages 99% of the time.

Using the values from Table 1 and Table 2 and 12½% desired underweight, we find from formula (1) $\sigma' = \frac{3.00}{2.534} = 1.18$

from formula (2) $\overline{X} = 1.15 \times 1.18 = 1.36$
since the data are in units of 5 grams we now know that we must have an average overweight of 6.8 grams or nearly one-fourth ounce for each five-pound package produced.

For a 100,200 pound car of five-pound packages this means that the company must give away 300 pounds of sugar in order to conform to the regulation. If the equipment or lack of control of package weights is such that the giveaway is more than 300 pounds, the economics of the situation are readily apparent. If, on the other hand, too many underweight packages are produced in this lot, you are in trouble with the FDA and again an expensive situation develops.

Consequently, the answer lies in being able to control this figure within reasonable limits. To do this a Control Chart is set up for the purpose of recording test results and to give the operator a basis upon which machine adjustments can be made.

Control Chart Limits

A Control Chart consists of two parts: One upon which the average of a sample set is plotted and the other upon which the range of the set is plotted (Figure 1).

Upper and lower control limits are established for the averages based on the scale information in Table 1 and upon the average overweight determined. These limits designated as three sigma (3σ) values encompass all chance results in over 99% of the trials providing there has been no shift in the average or some outside influence has not affected the mechanism. A 3σ upper limit is also calculated for the range section of the Control Chart.

By using information already obtained, i.e., the average range and $\overline{X}$, the control limits are calculated as follows using values from Table 2:

Control Limits for Average

Upper Control Limit (UCL) $= \overline{X} + A_2 \overline{R}$

where A_2 corresponds to the factor for 6 packages comprising the sample

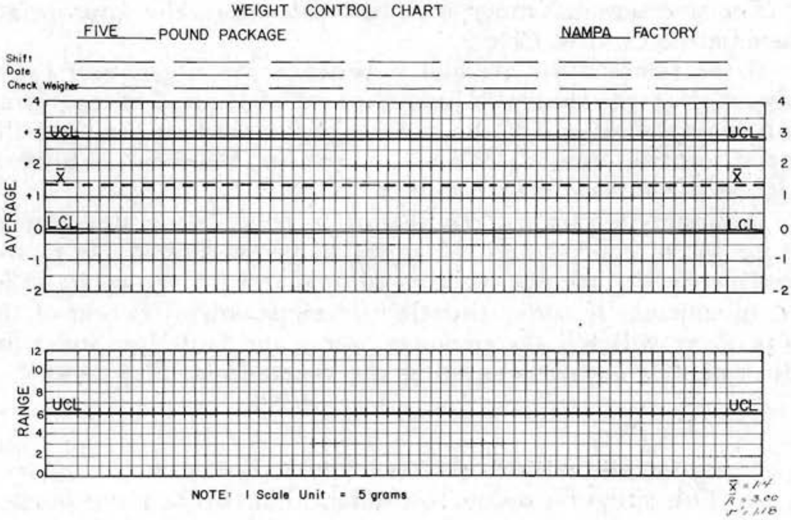

Figure 1.

Lower Control Limit for Average (LCL) $= \overline{X} - A_2 \overline{R}$.

The Upper Control Limit for the range is equal to $D_4 \overline{R}$ where D_4 corresponds to six measurements per set.

These calculations, therefore, give us by substituting the predetermined figures:

For Average

$$UCL = 1.36 + 0.483\ (3.00) = 2.8 \text{ (units)}$$

$$LCL = 1.36 - 0.483\ (3.00) = -0.1 \text{ (units)}$$

For Range

$$UCL = 3.00 \times 2.004 = 6.00 \text{ (units)}$$

The $\overline{X}$, UCL and LCL lines are drawn on the Control Chart on the Average section. The UCL for Range is also drawn in.

The Control Chart is now ready for use in actual testing and control of package weights.

Use of Control Chart and Interpretation of Results

A suitable work sheet should be made up on which can be shown the plus or minus values for each package comprising a sample set. Provision should be made for the total, average and range with an extra line to be used to indicate adjustments, if required.

A sample is withdrawn from the production line consisting of one package from each scale bucket in sequence. The packages are individually weighed on the check scales and the over or under weight in units is recorded. The sum, average and range are calculated. Identity of the individual packages and the corresponding bucket must be maintained.

The average and range are then plotted in the appropriate place on the Control Chart.

If the plot for the average is between the upper and lower control lines the system is judged to be in control or operating in a normal fashion. If the range plot is between zero and the upper control limit (UCL) the spread between individual buckets is assumed to be normal.

It should be pointed out that it is possible for the average to be *out* of control and the range to be *in* control. It is also possible for the average to be *in* control and for the range to be *out* of control. If either situation develops, an inspection of the work sheet will tell the operator where the fault lies and what adjustment is required to bring the system back into control.

For example:

Range In Control, Average Out of Control

This situation points to the fact that two or more buckets are weighing either too heavy or too light depending upon the location of the average plot. The buckets at fault are inspected and adjusted. Note of the change is recorded on the work sheet and another sample set is taken after the operator is satisfied the weighing response to the adjustment has stabilized (usually within 2 - 3 cycles). Results of the next sample are plotted to determine whether or not the correction step was sufficient to bring the system back into control.

Average In Control, Range Out of Control

In this case there is usually one one or possibly two buckets which have gone out of control in opposite directions. The work sheet will show what corrective action is required. A notation is made and another sample is taken.

If both range and average plots are within the control limit lines, no adjustment is made on any of the buckets. The package line is known to be performing normally. There is only one chance in a hundred that a sample set will indicate lack of control when actually the system is in control. The following discussion points to an exception to this rule and is part of the interpretation of results which should give all concerned additional confidence in Control Chart Technique.

After the Control Chart method has been in operation for only a short time, the graphical trends shown by lines connecting the consecutive sample plotting points yield a clear picture of the behavior of the packaging equipment.

If all of the average points fall below or above the $\overline{X}$ line but still between the LCL and UCL lines, there is an indication that the average of the production has shifted either down or upward. The result is that you are either producing more underweight units than desired or are giving away more sugar than is required to meet the weight regulations.

The appropriate small adjustment is then made to correct this small but significant trend. A well-controlled system will reveal points falling both above and below the $\overline{X}$ line in a sine wave pattern. No more than three to five points in succession should be all below or all above the X line.

Sampling Frequency and Personnel

The frequency of sampling depends upon and governs the degree of accuracy you wish to maintain. A sample should always be taken as soon as practical after the production line has started up after a shutdown or at the beginning of a shift. Samples should then be taken at thirty-minute or not more than sixty-minute intervals thereafter during the shift. Time must also be allowed for additional sampling after an adjustment has been made.

We have found that additional labor is not required to carry out a weight control program such as this. It is preferable to appoint one man on each shift who is thoroughly familiar with the machinery and operation to be responsible for the check weighing. This should be the primary job of the employee.

All foremen, and other supervisors should be familiar with the program and understand the objectives and interpretation of the results. As an additional aid to supervisors it has been recommended that the weight Control Chart on a particular package be continuous even though the production is intermittent. Notation as to date and shift can be made above the plotting for a particular time interval of operation. If more than one crew is used on production, the graphic plotting for the first shift can be in red pencil, second shift, blue and third shift, yellow.

It has been our experience that wherever Control Chart systems are used that all personnel involved take more interest in maintaining good weights of packages and better maintenance of equipment is achieved.

In addition, should an inspection of the plant be made by FDA representatives, or should an adverse situation arise, a good weight control system presented as evidence will go a long way toward assuring the public that you are endeavoring to maintain satisfactory weights in their behalf.

References

(1) DAVIES, O. L., "Statistical Methods in Research and Production", 3rd Edition, Oliver & Boyd, London, England; Hafner Publishing Company, New York.

(2) GRANT, E. L., "Statistical Quality Control", McGraw-Hill Book Company, New York.

(3) MORONEY, M. J., "Facts From Figures", A 236, Penguin Books, Ltd., 3300 Clipper Mill Road, Baltimore 11, Md.

(4) REYNOLDS, JOHN E., "A Practical Approach to a Weight Control Program", (in 3 parts), Package Engineering, Nov. and Dec. 1958 and Jan. 1959.

(5) Handbook, "Symbols, Definitions, and Tables for Industrial Statistics and Quality Control", Rochester Institute of Technology, Compiled by Industrial Statistics Committee, Eastman Kodak Company.