Estimating General Combining Ability From An Incomplete Crossing System[1]

GEORGE A. MILLIKEN, H. L. BUSH, ALVIN W. ERICKSEN, AND AKIO SUZUKI[2]

Received for publication May 25, 1970

Introduction

The estimation of general combining ability (GCA) when the crossing system is incomplete is a very difficult computational procedure without the use of a computer. The crossing system is usually such that a certain group of good or proven parents occurs more frequently than do unproven or new parents resulting in a greatly unbalanced array. To obtain estimates and confidence intervals for the GCA's, the crossing array is considered as an unbalanced two-way classification model in which male parents or pollinators are columns and female parents are rows. An exact analysis of the crossing system is obtained by solving the normal equations for the model by a matrix inversion technique from Graybill[3] which provides the analysis of variance, GCA estimates, confidence intervals for individual GCA's, and confidence intervals on the difference between GCA's for every pair of females and males. The confidence intervals provided LSD type inference procedures. All of the computations are easily obtained using a computer program.

The Crossing System

The complete diallel crossing system results from all possible crosses between lines where each is used as a male and as a female. In using male-sterile lines as female parents in a hybrid sugar beet program, it was of interest only to cross chosen male-sterile lines with a set of pollinators; thus reciprocal crosses were not considered. The crossing system used was a rectangular array consisting of a corner of a complete diallel crossing system with no diagonal elements included, as shown in Figure 1.

At the Great Western Agricultural Experiment Station, we were interested in evaluating a large number of parental lines. The data from the many crosses which occur in several experi-

---

[1] Contribution No. 138 of the Department of Statistics and Computer Science, Kansas Agricultural Experiment Station.

[2] Assistant Professor, Department of Statistics and Computer Science, Kansas State University, Manhattan, Kansas 66502, formerly Agronomist-Statistician; Head Plant Breeder, and plant breeder, respectively, Great Western Agricultural Experiment Station, Longmont, Colorado. 80501.

[3] Numbers in parentheses refer to literature cited.

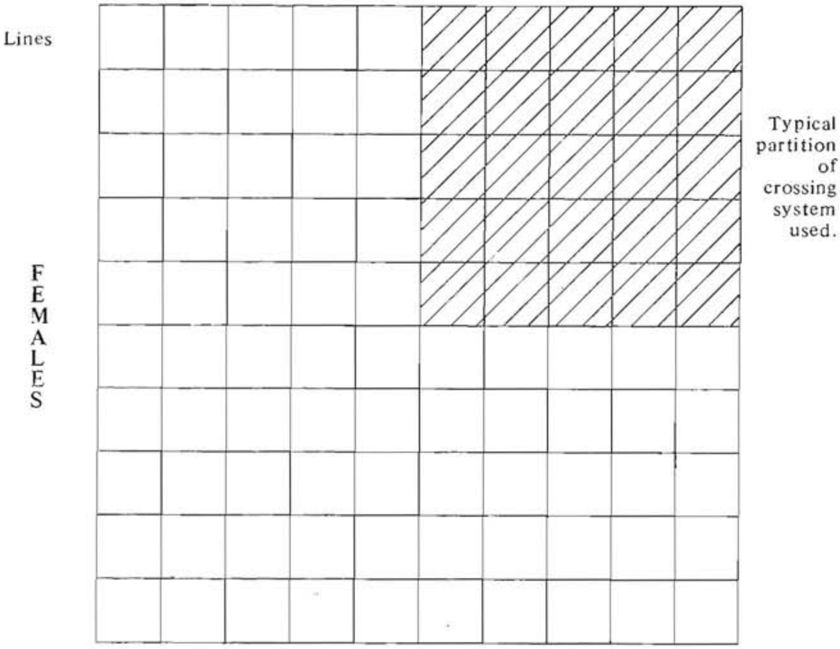

Figure 1.—Full diallel crossing system with shaded area showing the part of the crossing system studied.

ments at one location were combined to estimate the general combining abilities. All crosses did not occur in the same experiment, and to eliminate as much between experiment variation as possible, a common check variety was included in each experiment. The characteristics studied for each cross within an experiment was expressed as the percentage of the corresponding characteristics of the check variety in the experiment. The percentages from crosses occurring in several experiments were included in the crossing system, often resulting in 50 or more pollinators and 50 or more female lines. A complete crossing system would require 2500 or more crosses which is usually impossible to obtain. It has been found that a fraction of the possible crosses can provide reliable estimates of the GCA's. As a rule of thumb, at least 10% of all possible crosses should be included for meaningful results and a larger percentage of the possible crosses must be included as the number of pollinators and female lines included in the study decrease.

For example, an experiment with 5 pollinators and 5 female lines requires at least 10 crosses before the system can be analyzed (limitation on degrees of freedom) and more than 15 crosses are desirable (to obtain sufficient degrees of freedom to estimate error).

As a final restriction on the crossing system, the crosses must form a connected block-treatment array, as defined in Graybill (1961), in which female lines are considered as blocks and pollinators are treatments.

The Analysis

A particular cross can be included in the system more than once, thus, as shown in Figure 2a, the array of data considered was the sums for crosses and marginal sums for each pollinator and female line. Figure 2b shows a 5x5 crossing system, the data being recoverable sugar yields from a 1967 experiment. Here each cross was observed once or not at all.

POLLINATORS

FEMALES	1	2				t	SUMS
1	$y_{11\cdot}$	$y_{12\cdot}$	·	·	·	$y_{1t\cdot}$	$y_{1\cdot\cdot}$
2	$y_{21\cdot}$	$y_{22\cdot}$	·	·	·	$y_{2t\cdot}$	$y_{2\cdot\cdot}$
·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·
b	$y_{b1\cdot}$	$y_{b2\cdot}$	·	·	·	$y_{bt\cdot}$	$y_{b\cdot\cdot}$
SUMS	$y_{\cdot 1\cdot}$	$y_{\cdot 2\cdot}$	·	·	·	$y_{\cdot t\cdot}$	$y_{\cdot\cdot\cdot}$

Figure 2a.—Incomplete crossing systems with cell totals and marginal totals. The dot notation denotes the particular subscript has been summed over.

POLLINATORS

FEMALES	1	2	3	4	5	SUM
1	124.2	128.0			108.0	360.2
2	112.9	115.6			107.9	336.4
3	112.6		129.2	93.1		334.9
4	122.6	133.6	142.6			398.8
5	118.4	121.6	133.9		107.5	481.4
SUM	590.7	498.8	405.7	93.1	323.4	1911.7

Figure 2b.—Array showing observed data and marginal totals.

The model used to analyze the crossing system is

(1) $$y_{ijk} = \mu + \beta_i + \tau_j + e_{ijk}$$

$$i = 1,...,b; j = 1,...,t; k = 0,1,...n_{ij};$$

where μ is the general mean, β_i is the GCA of the ith female line, τ_j is the GCA of the jth pollinator and n_{ij} may be any finite integer, 0,1,2,... . Figure 3a contains the array of n_{ij}'s

where $n_{i\cdot} = \sum_{j=1}^{t} n_{ij}$; $n_{\cdot j} = \sum_{i=1}^{b} n_{ij}$; and $n_{\cdot\cdot} = \sum_{i=1}^{b}\sum_{j=1}^{t} n_{ij}$.

POLLINATORS

FEMALES	1	2	.	.	.	t	SUM
1	n_{11}	n_{12}	.	.	.	n_{1t}	$n_{1\cdot}$
2	n_{21}	n_{22}	.	.	.	n_{2t}	$n_{2\cdot}$
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
b	n_{b1}	n_{b2}	.	.	.	n_{bt}	$n_{b\cdot}$
SUM	$n_{\cdot 1}$	$n_{\cdot 2}$	.	.	.	$n_{\cdot t}$	$n_{\cdot\cdot}$

Figure 3a.—Incomplete crossing system with the number of observations for each cross and the totals for each female and pollinator.

POLLINATORS

FEMALES	1	2	3	4	5	SUM
1	1	1	0	0	1	3
2	1	1	0	0	1	3
3	1	0	1	1	0	3
4	1	1	1	0	0	3
5	1	1	1	0	1	4
SUM	5	4	3	1	3	16

Figure 3b.—Array of the number of times each cross was observed with female and pollinator totals.

The n_{ij} values for the example are included in Figure 3b.

The normal equations for the model in (1) are

$$\begin{aligned} &\mu : n_{\cdot\cdot}\hat{\mu} + \sum_i n_{i\cdot}\hat{\beta}_i + \sum_j n_{\cdot j}\hat{\tau}_j = y_{\cdot\cdot\cdot} \\ (2) \qquad &\beta_r : n_{r\cdot}\hat{\mu} + n_{r\cdot}\hat{\beta}_r + \sum_j n_{rj}\hat{\tau}_j = y_{r\cdot\cdot} \quad r=1,\ldots,b \\ &\tau_s : n_{\cdot s}\hat{\mu} + \sum_i n_{is}\hat{\beta}_i + n_{\cdot s}\hat{\tau}_s = y_{\cdot s\cdot} \quad s=1,\ldots,t \end{aligned}$$

where "^" denotes the estimator. The parameters we want to estimate are the effects of the pollinators and the female lines. Since the crossing system is incomplete, i. e., each pollinator does not occur with each female line, the desired estimates of the pollinators' general combining abilities are those adjusted for the female lines with which they were crossed. For example, suppose a particular pollinator occurs only with inferior female lines. The unadjusted mean or average of the pollinator is not necessarly a measure of the pollinator's effect on the population of female lines. The pollinator mean is thus adjusted as to indicate the possible GCA if the pollinator had been crossed with all the female lines. This adjustment is accomplished as the normal equations are solved. To obtain estimators of the GCA's for female lines adjusted for the pollinators with which they were crossed and the GCA's for pollinators adjusted for the female lines with which they were crossed, the normal equations were reduced to two sets of equations, a set containing only the β_r parameters and a set containing only the τ_s parameters.

To estimate the τ_s parameters, compute the quantities

$$(3) \qquad q_s = y_{\cdot s \cdot} - \sum_{i=1}^{b} n_{is} \bar{y}_{i \cdot \cdot} \quad s=1,\ldots,t$$

where $\bar{y}_{i\cdot\cdot} = \frac{y_{i\cdot\cdot}}{n_{i\cdot}}$ and compute the elements of the matrix

$\underline{A} = ((a_{rs}))$ where

$$(4) \qquad a_{ss} = n_{\cdot s} - \sum_{i=1}^{b} \frac{n^2_{is}}{n_{i\cdot}} \quad \text{and} \quad a_{rs} = -\sum_{i=1}^{b} \frac{n_{is} n_{ir}}{n_{i\cdot}} \quad r \neq s.$$

Figures 4 and 5 are respectively the $\underline{q}$ vector and the $\underline{A}$ matrix for the example. Estimates of the GCA's for pollinators are the solutions for the $\hat{\underline{\tau}}$ vector from the system of equations $\underline{A}\hat{\underline{\tau}} = \underline{q}$.

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} -6.4167 \\ 13.3167 \\ 40.7833 \\ -18.5333 \\ -29.1500 \end{bmatrix} \qquad q_s = y_{\cdot s \cdot} - \sum_{i=1}^{5} n_{is} \bar{y}_{i\cdot\cdot}$$

Figure 4.—The computed $\underline{q}$ vector for GCA estimates of pollinators adjusted for females.

$$\underline{A} = \begin{bmatrix} 3.4167 & -1.2500 & -0.9167 & -0.3333 & -0.9167 \\ -1.2500 & 2.7500 & -0.5833 & 0.0000 & -0.9167 \\ -0.9167 & -0.5833 & 2.0833 & -0.3333 & -0.2500 \\ -0.3333 & 0.0000 & -0.3333 & 0.6667 & 0.0000 \\ -0.9167 & -0.9167 & -0.2500 & 0.0000 & 2.0833 \end{bmatrix}$$

$$a_{ss} = n_{\cdot s} - \sum_{i=1}^{5} \frac{n^2_{is}}{n_{i\cdot}} \qquad a_{rs} = -\sum_{i=1}^{b} \frac{n_{is} n_{ir}}{n_{i\cdot}}$$

Figure 5.—The computed $\underline{A}$ matrix for GCA estimates of pollinators adjusted for females.

To estimate the β_r parameters, compute the quantities

$$(5) \qquad p_r = y_{r\cdot\cdot} - \sum_{j=1}^{t} n_{rj} \bar{y}_{\cdot j \cdot} \quad r=1,\ldots,b$$

and compute the elements of the matrix $\underline{B} = ((b_{rs}))$ where

$$(6) \qquad b_{rr} = n_{r\cdot} - \sum_{j=1}^{t} \frac{n^2_{rj}}{n_{\cdot j}} \text{ and}$$

$$b_{rs} = - \sum_{j=1}^{t} \frac{n_{rj} n_{sj}}{n_{\cdot j}} \qquad r \neq s.$$

Figures 6 and 7 are, respectively, the $\underline{p}$ vector and the $\underline{B}$ matrix for the example. Estimates of the GCA's for female lines are the solutions for the $\underline{\hat{\beta}}$ vector from the system of equations $\underline{B}\underline{\hat{\beta}} = \underline{p}$.

$$\underline{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} 9.5600 \\ -14.2400 \\ -11.5733 \\ 20.7267 \\ -4.4733 \end{bmatrix} \qquad p_r = y_{r\cdot\cdot} - \sum_{j=1}^{5} n_{rj} \bar{y}_{\cdot j\cdot}$$

Figure 6.—The computed $\underline{p}$ vector for GCA estimates of females adjusted for pollinators.

$$\underline{B} = \begin{bmatrix} 2.2167 & -0.7833 & -0.2000 & -0.4500 & -0.7833 \\ -0.7833 & 2.2167 & -0.2000 & -0.4500 & -0.7833 \\ -0.2000 & -0.2000 & 1.4667 & -0.5333 & -0.5333 \\ -0.4500 & -0.4500 & -0.5333 & 2.2167 & -0.7833 \\ -0.7833 & -0.7833 & -0.5333 & -0.7833 & 2.8833 \end{bmatrix}$$

$$b_{rr} = n_{r\cdot} - \sum_{j=1}^{5} \frac{n^2_{rj}}{n_{\cdot j}} \qquad b_{rs} = - \sum_{j=1}^{5} \frac{n_{rj} n_{sj}}{n_{\cdot j}}$$

Figure 7.—The computed $\underline{B}$ matrix for GCA estimates of females adjusted for pollinators.

In the above systems of equations the $\underline{A}$ and $\underline{B}$ matrices are singular and thus the inverses do not exist; therefore, to obtain solutions, add $\frac{1}{t}$ to each element of $\underline{A}$ to generate an $\underline{A}^*$ matrix and add $\frac{1}{b}$ to each element of $\underline{B}$ to generate a $\underline{B}^*$ matrix. The inverses of $\underline{A}^*$ and $\underline{B}^*$ exist and can be computed using any matrix inversion technique. Next, obtain a matrix $\underline{A}^+$ by subtracting $\frac{1}{t}$ from each element of $(\underline{A}^*)^{-1}$ and a matrix $\underline{B}^+$ by subtracting $\frac{1}{b}$ from each element of $(\underline{B}^*)^{-1}$. Figure 8 shows the $\underline{A}^*$, $(\underline{A}^*)^{-1}$ and $\underline{A}^+$ matrics and Figure 9 shows the $\underline{B}^*$, $(\underline{B}^*)^{-1}$, and $\underline{B}^+$ matrics for the 5x5 example. The estimates of the GCA's for pollinators are $\underline{\hat{\tau}} = \underline{A}^+\underline{q}$ and the estimates of the GCA's for the female lines are $\underline{\hat{\beta}} = \underline{B}^+\underline{p}$. These computations provide solutions which satisfy the

the restriction $\sum_{i=1}^{b} \beta_i = 0$ and $\sum_{j=1}^{t} \tau_j = 0$. Estimates of the GCA's for the example are in Figure 10.

$$\underline{A}^* = \begin{bmatrix} 3.6167 & -1.0500 & -0.7167 & -0.1333 & -0.7167 \\ -1.0500 & 2.9500 & -0.3833 & 0.2000 & -0.7167 \\ -0.7167 & -0.3833 & 2.2833 & -0.1333 & -0.0500 \\ -0.1333 & 0.2000 & -0.1333 & 0.8667 & 0.2000 \\ -0.7167 & -0.7176 & -0.0500 & 0.2000 & 2.2833 \end{bmatrix}$$

$$(\underline{A}^*)^{-1} = \begin{bmatrix} 0.4141 & 0.2195 & 0.1708 & -0.0076 & 0.2032 \\ 0.2195 & 0.5029 & 0.1519 & -0.1143 & 0.2401 \\ 0.1708 & 0.1519 & 0.5222 & 0.0465 & 0.1086 \\ -0.0076 & -0.1143 & 0.0465 & 1.2195 & -0.1441 \\ 0.2032 & 0.2401 & 0.1086 & -0.1441 & 0.5921 \end{bmatrix}$$

$$\underline{A}^+ \quad \begin{bmatrix} 0.2141 & 0.0195 & -0.0292 & -0.2076 & 0.0032 \\ 0.0195 & 0.3029 & -0.0481 & -0.3143 & 0.0401 \\ -0.0292 & -0.0481 & 0.3222 & -0.1535 & -0.0914 \\ -0.2076 & -0.3143 & -0.1535 & 1.0195 & -0.3441 \\ 0.0032 & 0.0401 & -0.0914 & -0.3441 & 0.3921 \end{bmatrix}$$

Figure 8.—The computed $\underline{A}^*$, $(\underline{A}^*)^{-1}$, and $\underline{A}^+$ matrices for estimating the GCA of the pollinators adjusted for females.

$$\underline{B}^* = \begin{bmatrix} 2.4167 & -0.5833 & 0.0000 & -0.2500 & -0.5833 \\ -0.5833 & 2.4167 & 0.0000 & -0.2500 & -0.5833 \\ 0.0000 & 0.0000 & 1.6667 & -0.3333 & -0.3333 \\ -0.2500 & -0.2500 & -0.3333 & 2.4167 & -0.5833 \\ -0.5833 & -0.5833 & -0.3333 & -0.5833 & 3.0833 \end{bmatrix}$$

$$(\underline{B}^*)^{-1} = \begin{bmatrix} 0.5045 & 0.1712 & 0.0541 & 0.1149 & 0.1554 \\ 0.1712 & 0.5045 & 0.0541 & 0.1149 & 0.1554 \\ 0.0541 & 0.0541 & 0.6487 & 0.1224 & 0.1149 \\ 0.1149 & 0.1149 & 0.1284 & 0.4915 & 0.1503 \\ 0.1554 & 0.1554 & 0.1149 & 0.1503 & 0.4240 \end{bmatrix}$$

$$\underline{B}^+ = \begin{bmatrix} 0.3045 & -0.0288 & -0.1459 & -0.0851 & -0.0446 \\ -0.0288 & 0.3045 & -0.1459 & -0.0851 & -0.0446 \\ -0.1459 & -0.1459 & 0.4487 & -0.0716 & -0.0851 \\ -0.0851 & -0.0851 & -0.0716 & 0.2915 & -0.0497 \\ -0.0446 & -0.0446 & -0.0851 & -0.0497 & 0.2240 \end{bmatrix}$$

Figure 9.—The computed $\underline{B}^*$, $(\underline{B}^*)^{-1}$, and $\underline{B}^+$ matrices for estimating the GCA of the females adjusted for the pollinators.

Table 1.—Analysis of variance.

Source	Degrees of freedom	Sums of squares	Mean Square	F Ratio
Pollinators GCA ADJ for females	$t-1=4$	$\sum \hat{\tau}_j q_j = 1394.8228$	348.706	35.328
Female GCA ADJ for pollinators	$b-1=4$	$\sum \hat{\beta}_i p_i = 320.8448$	80.2112	8.126
ERROR	$n_{..}-b-t+1 = 7$	$\sum y_{ijk}^2 - \frac{\sum y_{i..}^2}{n_{i.}} - \sum \hat{\tau}_j q_j = \sum y_{ijk}^2 - \frac{\sum y_{.j.}^2}{n_{.j}} - \sum \hat{\beta}_i p_i = 69.0939$	9.8706	

The analysis of variance table (Table 1) can now be constructed by computing the sum of squares due to pollinator GCA's adjusted for female lines, the sum of squares due to female line GCA's adjusted for pollinators and the sum of squares due to error.

The ratios of the mean square due to pollinator GCA's to mean square error and of the mean square due to female line GCA's to the mean square error provided tests of the hypotheses that the pollinators' GCA's are equal and that the female line GCA's are equal. The ratios for the example are in Table 1 indicating that the hypotheses are both false.

$$\underline{\hat{\tau}} = \underline{A}^+\underline{q} = \begin{bmatrix} 1.448 \\ 6.604 \\ 18.194 \\ -17.979 \\ -8.266 \end{bmatrix} \qquad \underline{\hat{\beta}} = \underline{B}^+\underline{p} = \begin{bmatrix} 3.446 \\ -4.488 \\ -5.613 \\ 7.492 \\ -0.837 \end{bmatrix}$$

Figure 10.—Estimates of the GCA for pollinators ($\underline{\hat{\tau}}$) and for females ($\underline{\hat{\beta}}$).

If the hypotheses seem to be false, as for the example, the next step is to determine the superior lines. This can be accomplished in two ways, (1) compute confidence intervals for each $\mu + \tau_j$ and $\mu + \beta_i$, or (2) compute confidence intervals for each difference $\tau_i - \tau_j$ and $\beta_i - \beta_j$. The $(1\text{-}\alpha)$ 100% confidence interval on $\mu + \tau_j$ is

$$(7) \qquad \hat{\mu} + \hat{\tau}_j - t_{\alpha/2}(m)\sqrt{EMS}\sqrt{a^+_{jj}} \leq \mu + \tau_j \leq \hat{\mu} + \hat{\tau}_j + t_{\alpha/2}(m)\sqrt{EMS}\sqrt{a^+_{jj}}$$

where a^+_{jj} is the jth diagonal element of the $\underline{A}^+$ matrix and $m = n_{..}-b-t+1$. The $(1\text{-}\alpha)$ 100% confidence interval on $\mu + \beta_i$ is

$$(8) \qquad \hat{\mu} + \hat{\beta}_i - t_{\alpha/2}(m)\sqrt{EMS}\sqrt{b^+_{ii}} \leq \mu + \beta_i \leq \hat{\mu} + \hat{\beta}_i + t_{\alpha/2}(m)\sqrt{EMS}\sqrt{b^+_{ii}}$$

where b^+_{ii} is the ith diagonal element of the $\underline{B}^+$ matrix. The $\hat{\mu} + \tau_j$

are the adjusted means for pollinators and the $\mu + \beta^+_i$ are the adjusted means for female lines. The variances of the adjusted means are $EMSxa^+_{jj}$ for pollinators and $EMSxb^+_{ii}$ for female lines. Table 2 shows means, variances, and 95% confidence intervals for the pollinator means $(\mu + \tau_j)$ and female line means $(\mu + \beta_i)$ of the example. A simultaneous inference procedure is to say two GCA's are different if the confidence intervals about their means do not overlap.

Table 2.—The computed adjusted means, variances of the adjusted means, and 95% confidence intervals on the $\mu + \tau_j$ and $\mu + \beta_i$.

POLLINATORS	UNADJ MEAN	ADJ MEAN	VARIANCE	LOW CI	UPPER CI
1	118.140	118.088	2.113	117.49	124.37
2	124.700	123.245	2.990	121.99	130.17
3	135.233	134.834	3.180	133.46	141.89
4	93.100	98.661	10.003	94.00	109.00
5	107.800	108.374	3.870	106.56	115.87
FEMALES	UNADJ MEAN	ADJ MEAN	VARIANCE	LOW CI	UPPER CI
1	120.067	122.875	3.006	118.83	127.03
2	112.133	114.942	3.006	110.89	119.09
3	111.633	113.836	4.428	108.89	118.84
4	132.933	126.922	2.878	122.96	130.98
5	120.350	118.592	2.211	115.13	122.16

The second procedure is to compute confidence intervals on the differences $\tau_i - \tau_j$ and $\beta_i - \beta_j$. The variance of the difference $\hat{\tau}_i - \hat{\tau}_j$ is

(9) $c_{ij} = (a^+_{ii} + a^+_{jj} + 2a^+_{ij}) \times EMS$, where a^+_{ij} are elements of $\underline{A}^+$.

The variance of the difference $\hat{\beta}_i - \hat{\beta}_j$ is

(10) $d_{ij} = (b^+_{ii} + b^+_{jj} + 2b^+_{ij}) \times EMS$, where the b^+_{ij} are elements of $\underline{B}^+$.

A $(1\text{-}\alpha)$ 100% confidence interval for $\tau_i - \tau_j$ is

(11) $\hat{\tau}_i - \hat{\tau}_j - t_{\alpha/2}(m)\sqrt{c_{ij}} \leq \tau_i - \tau_j \leq \hat{\tau}_i - \hat{\tau}_j + t_{\alpha/2}(m)\sqrt{c_{ij}}$

and a $(1\text{-}\alpha)$ 100% confidence interval for $\beta_i - \beta_j$ is

(12) $\hat{\beta}_i - \hat{\beta}_j - t_{\alpha/2}(m)\sqrt{d_{ij}} \leq \beta_i - \beta_j \leq \hat{\beta}_i - \hat{\beta}_j + t_{\alpha/2}(m)\sqrt{d_{ij}}.$

The simultaneous inference procedure is to say two GCA's are different if the confidence interval on their difference does not contain zero.

The second procedure produces somewhat shorter confidence intervals, thus enabling more differences to be directed. This is seen by comparing the confidence intervals on the differences in Table 3 where one additional difference is shown over that by comparing overlapping confidence intervals in Table 2.

Table 3.—Estimates and 95% confidence intervals on the difference of pollinators and females.

POLLINATORS

DIFFERENCE	ESTIMATE OF DIFFERENCE	LOWER CI	UPPER CI
$\tau_1 - \tau_2$	−5.156	−10.282	−0.030
$\tau_1 - \tau_3$	−16.746	−22.463	−11.029
$\tau_1 - \tau_4$	19.427	9.907	28.947
$\tau_1 - \tau_5$	9.714	3.972	15.456
$\tau_2 - \tau_3$	−11.590	−17.887	−5.293
$\tau_2 - \tau_4$	24.583	14.227	34.939
$\tau_2 - \tau_5$	14.870	9.056	20.684
$\tau_3 - \tau_4$	36.173	26.653	45.693
$\tau_3 - \tau_5$	26.460	19.439	33.481
$\tau_4 - \tau_5$	−9.713	−20.456	1.030

FEMALES

DIFFERENCE	ESTIMATE OF DIFFERENCE	LOWER CI	UPPER CI
$\beta_1 - \beta_2$	7.934	1.880	13.988
$\beta_1 - \beta_3$	9.059	1.483	16.635
$\beta_1 - \beta_4$	−4.046	−10.537	2.445
$\beta_1 - \beta_5$	4.283	−1.330	9.896
$\beta_2 - \beta_3$	1.125	−6.451	8.701
$\beta_2 - \beta_4$	−11.980	−18.471	−5.489
$\beta_2 - \beta_5$	−3.651	−9.264	1.962
$\beta_3 - \beta_4$	−13.105	−22.036	−8.098
$\beta_3 - \beta_5$	−4.776	−11.583	2.031
$\beta_4 - \beta_5$	8.329	2.515	14.143

The above procedure is computationally difficult for a desk calculator, but it can be programmed for operation on a high speed computer. Programs are now available for the CDC 6400 and the 360/50 IBM computer systems. The computer programs solve the normal equations for the values of β_i and τ_j, compute the entries for the analysis of variance table, and compute the confidence intervals about the parameters $\mu + \tau_j$ and $\mu + \beta_i$. The program does not compute confidence intervals about the differences $\beta_i - \beta_j$ and $\tau_i - \tau_j$ as the required number of pages of output becomes large.

The procedure described is not restricted to sugar beet data; the example is included only to demonstrate the procedure. The procedure can be used for any experimental situation producing an incomplete crossing system as described above. Still further, the procedure can be used to analyze any set of data from a two-way design without interaction.

Summary

A computational procedure is presented to estimate the general combining abilities (GCA) of the parents of an incomplete crossing system. The procedure also provides the analysis of variance table from which we can test the hypotheses; 1) no difference between the female line GCA's and 2) no difference between the pollinator GCA's. Confidence intervals can be constructed about the individual pollinator and female line means and about the difference of two GCA's. The confidence intervals are used as a LSD type inference procedure. Besides presenting the computational procedures, an example is discussed in detail to demonstrate the necessary calculations. A computer program is available for doing the described analysis.

Acknowledgement

The authors wish to acknowledge the original idea of Sydney Ellerton, Bush Johnson, Ltd., Woodham Mortimer Hall, Maldon, Essex, England, in which he made adjustments in an incomplete diallel crossing system and for his helpful advice during this investigation.

Literature Cited

GRAYBILL, F. A. 1961. Introduction to Linear Statistical Models, McGraw Hill, New York, Vol. 1.