Regression Analysis of Chemical Measurements on Hybrid Sugarbeets to Root Yield, Sugar Percentage and Impurity Index[1]

GEORGE K. RYSER, J. C. THEURER, AND JAY L. HADDOCK[2]

Received for publication February 24, 1975

Introduction

The relationship of chemical composition of leaf blades, leaf petioles, and beet pulp to root yield and quality in the hybrid sugarbeet has not been well defined.

Investigators have shown that certain inorganic constituents in sugarbeet roots reduce the recovery of crystallized sugar (1, 8, 13, 15, 16, 20).[3] Other scientists have suggested breeding methods for selection of lines low in inorganic factors such as Na, K, N, and Ca (3, 4, 5, 7, 18, 19). These studies generally resulted in a corresponding decrease in root yield and sugar percentage, which made them undesirable.

We believed that more information could be obtained by a regression study of root yield and sugar percentage with the quantity of undesirable inorganic constituents of these varieties. Without much extra cost, chemical analysis of leaf blades, plant petiole samples, or both could be made, as well as the chemical measurement on the pulp of the same plants.

A regression model-building program was developed to show the relative importance of chemical constituents to yield and quality factors of sugarbeet roots. Such a program should have general application in developing critical levels or threshold values for maximum yield and quality of sugarbeets (11, 12).

Materials and Methods

For a detailed chemical study, 18 hybrids were selected from variety tests conducted in 1966. Several of these hybrids had common male and female parents. Three varieties were used as checks, two of which were commercial sugarbeet company varieties and the third an experimental hybrid developed at Logan, Utah. Details of the cultural practices on this test are described elsewhere (10).

---

[1]Cooperative investigations of the Agricultural Research Service, U. S. Department of Agriculture; the Beet Sugar Development Foundation; and the Utah State Agricultural Experiment Station. Approved as Journal paper No. 1895, Utah Agricultural Experiment Station, Logan, Utah, 84322.

[2]Research Agronomist and Research Geneticist, Agricultural Research Service, U. S. Department of Agriculture, Crops Research Laboratory, Utah State University; and former Research Soil Scientist, Soil and Water Conservation Service, respectively.

[3]Numbers in parentheses refer to literature cited.

To have the plants under highest possible stress for nutrient absorption, leaf and petiole samples were obtained in August (the period of most rapid growth). Eighteen recently matured blades and petioles were selected from every other plant in the plot. They were washed in deionized water, dried in a forced-draft oven at 70°C, and ground in a stainless steel mill to pass a 40-mesh screen.

One gram of finely ground leaf petioles was extracted with 100 ml of 2% acetic acid solution. P was determined by Barton's method (2). Na and K were determined on diluted aliquots by flame photometry. Soluble N was obtained by the micro-Kjeldahl method, with 20 ml of solution. Nitrate N was obtained from the extract by the spot-plate diphenylamine-color method.

Leaf blades were wet-digested with HNO_3, followed by $HClO_4$ acid, after the method of Gerritz (9). Ca, Mg, Cu, Zn, Mn, and Fe were determined by means of the Perkin-Elmer[4] atomic absorption spectrophotometer. Total N was obtained on 0.2 g of finely ground (40-mesh) oven-dried plant material by the micro-Kjeldahl digestion distillation and titration method. S was determined from an aliquot of the acid digest according to Jackson's turbidimetric method (14). Cl was determined from a water extract of the oven-dried, ground plant material by potentiometric measurement of Cl ions, with the Beckman 39048 silver electrode and the Beckman expanded-scale pH meter. B was determined from 0.1 g of ground leaf tissue by the curcumin colorimetric method outlined by Jackson (14).

For the chemical determinations on dried pulp, the beet pulp was dried at 70°C in a forced-draft oven and ground in a stainless-steel mill to pass a 40-mesh screen. Total N, P, K, Na, Mn, Cu, Mg, Zn, S, Fe, Ca, and B were determined.

Na and K composition of rasped beet pulp was obtained by flame photometry on an aliquot of the leaded clear extract. Amino-N content of rasped beet pulp was determined by a modification of the Stanek-Pavlas copper reagent method.

The variables used as X_i independent are coded in the following order throughout the study: (a) from dry pulp X_i: $i = 1$ to 12; total N, P, K, Na, Mn, Cu, Mg, Zn, S, Fe, Ca, and B; (b) from leaf blades X_i: $i = 13$ to 25, total N, P, K, Na, Mn, Cu, Mg, Zn, S, Fe, Ca, B, and Cl; (c) from petiole X_i: $i = 26$ to 29, soluble N, P, K, and Na; and (d) from wet pulp X_i: $i = 30$ to 32, amino-N, K, and Na. For the dependent Y_k variables, Y_k: $k = 1$ to 5, where Y_1 = percentage of dry matter in dry pulp, Y_2 = gross sugar, Y_3 = tons per acre, Y_4 = sugar percentage, and Y_5 = impurity index, calculated from wet pulp measurements.

The quantity-quality factors, as described by Haddock and Stuart (11) for dry pulp, leaf blades, and petioles, were calculated as QF = N +

[4]Mention of a trademark or proprietary product does not constitute a guarantee or warranty of the product by the U. S. Department of Agriculture, and does not imply its approval to the exclusion of other products that may also be suitable.

P + K in me/100 g, QN = percentage of total N in QF, QP = percentage of P in QF, and QK = percentage of K in QF.

The simple correlation matrix obtained for the multiple regression models was studied for a better understanding of the relationships between variables. Three-dimensional graphs were constructed so we could observe the relationship between pairs of variables and the Y_k dependent variables. The regression method was a computer program used by the senior author and developed by Dr. Rex Hurst, Applied Statistics and Computer Science Department Head, Utah State University, Logan (17).

Linear models of 29 independent variables were calculated with X_1 to X_{12} on dry pulp, X_{13} to X_{25} on blades, and X_{26} to X_{29} on petioles with four of the dependent Y_k variables, Y_2, Y_3, Y_4, and Y_5. These models were repeated with the quantity-quality factors on the dry pulp, blade, and petiole measurements.

Models for each plant material—dry pulp and leaf blade—were calculated by the independent variables X_1 to X_{12} (dry pulp) and X_{13} to X_{24} (leaf blade), with each of the five dependent yield measures as follows: Y_1 = percentage of dry matter, Y_2 = gross sugar, Y_3 = tons per acre, Y_4 = sugar percentage, and Y_5 = impurity index. These models were also corrected as to the effect of variety levels and replication levels by the use of dummy variables (a covariance matrix adjustment applied to the model). Because N, P, K, and Na are the most generally measured elements at harvest on agronomic tests, separate models with only these elements included on each plant material were also studied.

Polynomials in the square and cubic form were developed for comparison with the linear models.

The stepwise method used is very similar to one described by Draper and Smith as the "Backward Elimination Procedure" (6). After the model with all variables has been calculated, the variable with the lowest F test is dropped, and another model is calculated. This process is repeated until all variables have been dropped. This method gives the experimenter a chance to see at what point the lowest error mean square is obtained for the most efficient model.

Results

The means for the 18 hybrids and their significance relative to the means of two commercial check varieties have been published by Haddock, et al. (10). The overall means for all independent X_i variables and the five dependent Y_k variables are given in Table 1. The coefficients of variablility are given because they bear on the R^2 values obtained in a regression analysis. The C. V.'s ranged from 4% for sugar percentage in wet pulp to 41% for Na in dry pulp (Table 1). These C. V.'s are not high for biological material such as in this test. It would have been better if the range had been increased by use of fertility levels.

Table 1.—Means* of independent (X_i) and dependent (Y_k) variables, with coefficient of variation.

Variable	Wet pulp (ppm)	C. V. (%)	Dry pulp (ppm)	C. V. (%)	Blades (ppm)	C. V. (%)	Petioles (ppm)	C. V. (%)
X_i Total N	200†	22.79	6,267	16.95	42,000	9.00	7,444	26.05
X_i P			751	27.82	2,408	11.79	1,484	17.92
X_i K	1,493	12.52	5,613	23.54	39,416	18.23	43,220	13.54
X_i Na	127	27.22	492	41.11	9,528	13.70	6,534	22.96
X_i Mn			29	24.46	148	20.11		
X_i Cu			4	30.52	12	21.43		
X_i Mg			1,411	9.29	8,682	8.87		
X_i Zn			8	35.97	25	21.19		
X_i S			311	15.15	4,981	15.19		
X_i Fe			51	40.52	154	33.35		
X_i Ca			2,034	19.39	14,451	19.39		
X_i B			113	11.97	63	10.91		
X_i Cl					1,819	16.70		
Quantity QF (me/100 g N+P+K)			66.37	14.56	424	6.87	178	12.80
Quality QN % of quantity			67.24	7.18	70.62	5.57	29.68	18.92
Quality QP % of quantity			11.10	19.90	5.51	10.84	8.13	17.97
Quality QK % of quantity			21.66	17.35	23.85	16.75	62.19	8.49
Y_1 Total dry matter (%)			21					
Y_2 Gross sugar (lb/a)	6,931	13.62						
Y_3 Tons/acre	23.26	13.07						
Y_4 Sugar percentage	14.90	4.09						
Y_5 Impurity index (ppm)	418	15.57						

*Means are based on 168 observations.

† Amino-N only.

Simple Correlations

Measurements on dry pulp of N, P, K, Na, Mg, and S are all essentially negatively correlated with percentage of dry matter (Y_1) and percentage of sugar (Y_4). These same elements were all positively correlated with impurity index (Y_5). Of the 60 possible correlations, 33 were significant at least at the 5% level (Table 2). On leaf-blade measurements, gross sugar (Y_2) was positively correlated with P, K, Cu, Zn, B, and Cl. Tons per acre (Y_3) was similar to gross sugar, with P, Cu, S, and B being

Table 2.—Simple correlations with dependent Y_k's and all independent X_i values.

Independent variable	Dependent Variable				
	Y_1 Dry matter (%)	Y_2 Gross sugar	Y_3 tons/acre	Y_4 Sugar (%)	Y_5 Impurity index
Dry Pulp					
1 = N	−0.304**	−0.118	0.024	−0.428**	0.676**
2 = P	−0.189*	0.298**	0.350**	0.085	0.158*
3 = K	−0.478**	0.101	0.269**	−0.452**	0.680**
4 = Na	−0.599**	−0.169*	0.054	−0.663**	0.622**
5 = Mn	0.187*	−0.055**	−0.019**	−0.237**	−0.439**
6 = Cu	−0.094	−0.034	0.011	−0.140	0.150
7 = Mg	−0.221**	−0.192*	−0.082	0.338**	0.203**
8 = Zn	0.003	0.141	−0.112	0.086	−0.021
9 = S	−0.373**	−0.084	0.068	−0.444*	0.659**
10 = Fe	0.090	0.051	−0.001	0.150	0.124
11 = Ca	−0.058	−0.332**	−0.284**	−0.206**	−0.056
12 = B	−0.058	−0.201**	−0.172*	−0.178*	−0.092
Leaf Blades					
13 = N	−0.037	0.086	0.107	−0.067	0.241**
14 = P	−0.121	0.555**	0.582**	0.089	0.188*
15 = K	0.067	0.152*	0.083	0.224*	−0.262**
16 = Na	0.146	−0.363**	−0.392**	0.040	−0.173*
17 = Mn	0.093	−0.066	−0.111	0.142	−0.222**
18 = Cu	−0.139	0.208**	0.248**	0.060	0.121
19 = Mg	0.082	−0.055	−0.059	0.010	0.064
20 = Zn	−0.158*	0.215**	−0.256**	−0.111	0.136
21 = S	0.023	−0.231**	0.248**	0.060	0.129
22 = Fe	0.123	0.144	−0.189*	0.106	−0.105
23 = Ca	0.193*	−0.266**	−0.324**	0.150	0.112
24 = B	−0.250**	0.274**	0.327**	−0.135	0.227**
25 = Cl	−0.171*	0.201**	−0.285**	0.201**	−0.163
Dry Pulp					
Quantity	-0.443*	−0.006	0.175*	−0.515**	0.805**
Quality (N)	0.127	0.252**	0.236**	−0.021	−0.024
Quality (P)	0.126	0.301**	0.226**	0.276**	−0.365**
Quality (K)	−0.265**	0.143	0.212**	−0.163*	0.221**
Leaf Blades					
Quantity	−0.007	0.259**	0.226**	0.117	0.044
Quality (N)	0.014	−0.099	−0.045	−0.182*	0.281**
Quality (P)	−0.124	0.464**	0.510**	−0.048	0.172*
Quality (K)	−0.066	0.074	0.005	0.220**	−0.312**

*, ** Means are significant at the 0.05 and 0.01 levels, respectively.

positively correlated and Na and Ca being negatively correlated. With impurity index (Y_5), N, P, and B showed positive correlations, and K, Na, and Mn showed negative ones. There seemed to be no consistent positive or negative relationship with Y_1, Y_2, Y_3, and Y_5, but there was an essentially positive one with Y_4. Of the 65 possible correlations for leaf blades, 30 were significant, at least at the 5% level (Table 2).

The quantity factor in dry pulp was significantly correlated with all Y_k factors except Y_2. These correlations were negative with Y_1 and Y_4 but positive with Y_3 and Y_5. These relationships did not hold for the blade measurements as the only significant correlations were positive with Y_2 and Y_3 (Table 2).

The Quality (N) and Quality (P) were both positively correlated with Y_2 and Y_3 for dry pulp but only Y_2 and Quality (P) were correlated in the blades.

Within the dependent Y_k measurements, Y_1 was correlated not at all with Y_2, but negatively with Y_3 and Y_5 and highly positively with Y_1 ($r = 0.85$). Y_2 was negatively correlated with Y_4 and Y_5 with an r value of -0.65.

The simple correlations between the 12 measurements on dry pulp and leaf blades with the four measurements on petioles are given in Table 3. In general P, Na, Mn, and S showed inverse correlations for dry pulp vs leaf blade measurements for petiole N and K. N, P, K, and Na in dry pulp and leaf blade are significantly positively correlated with the same element in the petiole.

Three-Dimensional Graphs

Three-dimensional graphs showing the effect of N and K on gross sugar, tons per acre, and impurity index are given in Fig. 1. The graph surfaces are similar for measurements of N and K on dry pulp, leaf blades, and petioles as per the rows of Fig. 1. The different dependent Y_k's can be compared by the columns of Fig. 1. The surfaces of all the the graphs vary in degree of slope, yet tend to be in the same direction.

Fig. 1G shows a probable curvilinear relationship between the dependent impurity index for units of N present; this curvilinear relationship is affected by the units of K. A regression model should include both linear terms, units of N and units of K, and the interactions (units of N times units of K and units of N^2 times units of K). This relationship seems reasonable for all the surfaces, except for those in Fig. 1B, 1E and 1H on blades. The surfaces obtained with the quantity-quality factors, though not shown, were similar.

There are as many three-dimensional graphs as correlation values in the 37×37 correlation matrix. A sampling of about 200 of these graphs gave results similar to those of Fig. 1. All graphs with Y_1 and Y_4 were very flat, because of low coefficients of variability.

Linear Regression Models

The first linear models were made up of the 12 measurements on dry pulp, 13 on blades, and four on petioles. The stepwise regression

Table 3.—Correlations between 12 chemical measurements on dry pulp and leaf blades with four measurements on petioles.

	Petiole							
	N		P		K		Na	
Variable	Dry pulp	Leaf blade	Dry pulp	Leaf blade	Dry pulp	Leaf blade	Dry pulp	Leaf blade
N	0.531**	0.482**	−0.079	−0.016	−0.339**	−0.210**	0.212**	0.098
P	−0.148*	0.092	0.440**	0.552**	−0.344**	0.224**	−0.407**	−0.416**
K	0.105	−0.330**	0.275**	0.136	0.231**	0.660**	0.149	−0.430**
Na	0.194*	0.004	−0.063	0.422**	−0.392**	0.448**	0.223*	0.668**
Mn	0.423**	−0.379**	0.113	−0.105	0.213**	0.188*	−0.230**	−0.001
Cu	0.018	0.065	0.033	0.211**	−0.035	0.122	0.037	−0.166*
Mg	0.157*	0.175*	0.055	0.297**	−0.285**	−0.271**	0.118	0.270**
Zn	−0.023	0.091	0.019	0.015	0.075	−0.067	−0.175*	−0.082
S	0.161*	−0.346**	0.112	0.224**	0.061	0.513**	0.075	−0.410**
Fe	0.116	−0.030	−0.048	−0.068	−0.139	−0.070	0.082	0.158*
Ca	0.145*	−0.069	−0.099	0.011	0.115	0.150*	0.173*	0.135
B	0.072	0.140	−0.160	0.245**	−0.125	0.234**	0.038	−0.291**

* = Significant at the 5% level.
**= Significant at the 1% level.

Table 4.—Stepwise regression of 12 elements on dry pulp, 13 on blades, and 4 on petioles on four Y_k measurements.

Variables in model (no.)	Y_2 = Gross Sugar		Y_3 = Tons/acre		Y_4 = Sugar Percentage		Y_5 = Impurity Index	
	R^2	Stepwise variable order	R^2	Stepwise variable order	R^2	Stepwise variable order	R^2	Stepwise variable order
1	0.3075	P(B)	0.3382	P(B)	0.4391	Na	0.4630	K
2	0.4011	Na(P)	0.4166	Na(P)	0.4949	K	0.7073	N
3	0.4386	Zn(B)	0.4752	Zn(B)	0.5336	B	0.7517	Na
4	0.4841	Ca	0.5050	Ca(B)	0.5553	P(P)	0.7593	P
5	0.4967	Ca(B)	0.5289	Ca	0.5760	Cl(B)	0.7680	Na(P)
6	0.5144	B	0.5425	B(B)	0.5821	N(P)	0.7738	Mg
7	0.5342	K	0.5556	B	0.5929†	N(B)	0.7818	N(P)
8	0.5455†	B(B)	0.5672†	N(P)	0.6008	Ca	0.7878	Cu
9	0.5541	Na	0.5747	K(P)	0.6056	Cu	0.7924	S
10	0.5631	K(B)	0.5810	Cl(B)	0.6103	S(B)	0.7929	S(B)
11	0.5720	Zn	0.5871	Mg	0.6138	Mg	0.8025	Mn
12	0.5774	Mg	0.5929	Zn	0.6183	Fe	0.8075†	Fe
13	0.5830	P(P)	0.5972*	Fe(B)	0.6214	Zn	0.8093*	Ca
14	0.5872	Cl(B)	0.5987	K(B)	0.6243*	Na(P)	0.8108	Zn(B)
15	0.5906	Fe(B)	0.6001	S(B)	0.6263	P	0.8116	Mg(B)
16	0.5931*	Mg(B)	0.6014	P(P)	0.6279	K(B)	0.8125	K(B)
17	0.5947	Fe	0.6028	Na	0.6289	N	0.8132	Zn
18	0.5960	Mn	0.6044	P	0.6296	Zn(B)	0.8134	Mn(B)
19	0.5975	K(P)	0.6057	Cu(B)	0.6302	P(B)	0.8136	N(B)
20	0.5982	N(P)	0.6068	Mn	0.6307	Mn	0.8139	B

(continued next page)

Table 4.—Continued.

Variables in model (no.)	Y_2 = Gross Sugar		Y_3 = Tons/acre		Y_4 = Sugar Percentage		Y_5 = Impurity Index	
	R^2	Stepwise variable order	R^2	Stepwise variable order	R^2	Stepwise variable order	R^2	Stepwise variable order
21	0.5994	N	0.6076	Fe	0.6312	B(B)	0.8140	K(P)
22	0.6002	Mn(B)	0.6084	N	0.6316	Mg(B)	0.8141	Cu(B)
23	0.6008	S(B)	0.6090	Mn(B)	0.6319	Na(B)	0.8142	P(P)
24	0.6014	P	0.6094	Mg(B)	0.6322	Fe(B)	0.8142	Cl(B)
25	0.6018	Na(B)	0.6099	N(B)	0.6322	K(P)	0.8143	Fe(B)
26	0.6020	Cu(B)	0.6103	Na(B)	0.6323	S	0.8143	P(B)
27	0.6022	Cu	0.6104	Cu	0.6323	Ca(B)	0.8143	Na(B)
28	0.6023	N(B)	0.6105	Cu	0.6323	Mn(B)	0.8143	B(B)
29	0.6023	S	0.6105	S	0.6323	Cu(B)	0.8143	Ca(B)

* Most efficient model
† All terms significant
(B) = Blade
(P) = Petiole

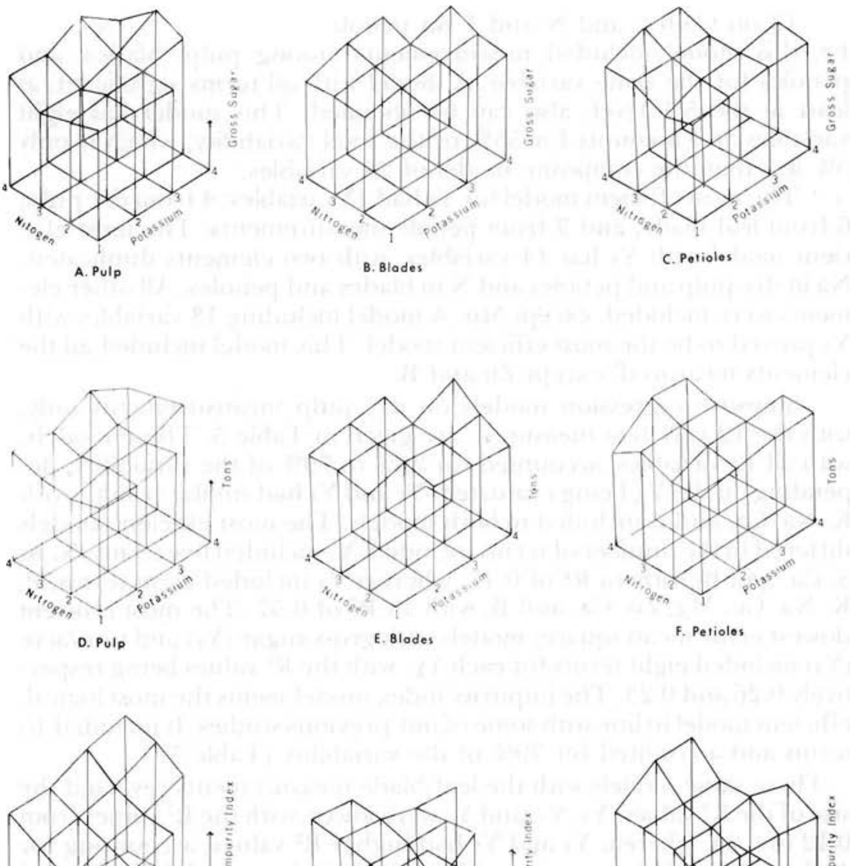

Figure 1.—Typical three-dimensional graphs for Y_2 (Gross sugar), Y_3 (tons per acre) and Y_5 (Impurity index) with pulp, blade, and petiole measurements.

order of the elements and R^2 values for each Y_k model is given in Table 4. This table is based on the computer output of the stepwise regression run for each Y_k. Data in the table is given in a reverse order of computer analysis because the output started with the complete model. The first term dropped for Y_2, gross sugar, was S, the next N (blade), and so forth, until the model with only one term, P (blade) was left.

For gross sugar, the most efficient model (model with the lowest error mean square) had 16 variables, with an R^2 of 0.59 (Table 4). These were K, Na, Mg, Zn, Ca, and B on dry pulp; P, K, Mg, Zn, Fe, Ca, B, and Cl on blades; and N and P on petioles. Except for N, Na, and Fe, this model included measurements among pulp, blades, and petioles for the same variable. A model with all terms significant, at least at the 5% level, also can be obtained. This model has eight variables and accounts for 55% of the total variability, which is only 5% less than the composite model of 29 variables.

The most efficient model for Y_3 had 13 variables: 4 from dry pulp, 6 from leaf-blade, and 3 from petiole measurements. The most efficient model with Y_4 has 14 variables, with two elements duplicated: Na in dry pulp and petioles and N in blades and petioles. All other elements were included, except Mn. A model including 13 variables with Y_5 proved to be the most efficient model. This model included all the elements measured, except Zn and B.

Stepwise regression models on dry pulp meansurements only, with the 12 variables measured, are given in Table 5. These models, with all 12 variables, accounted for 23% to 79% of the variability, depending on the Y_k being evaluated. Y_1 and Y_4 had similar results, with K, Na, Ca, and B included in both models. The most efficient models differed in the number of terms included. Y_1 included five terms, N, K, S, Ca, and B, with an R^2 of 0.47; whereas Y_4 included eight terms, P, K, Na, Cu, Mg, Zn, Ca, and B, with an R^2 of 0.57. The most efficient (lowest error mean square) models with gross sugar (Y_2) and tons/acre (Y_4) included eight terms for each Y_k, with the R^2 values being respectively 0.26 and 0.23. The impurity index model seems the most logical, efficient model in line with some of our previous studies. It included 10 terms and accounted for 79% of the variability (Table 5).

These same models with the leaf-blade measurements reversed the size of the R^2 values. Y_1, Y_4, and Y_5 were lower, with the R^2 values from 0.12 to 0.24, whereas Y_2 and Y_3 had higher R^2 values, accounting for 43% to 51% of the variation. The most efficient model for Y_2 and Y_3 each had the same seven terms but not in the same order (Table 5).

The next models were calculated with the four common measurements N, P, K, and Na only. These calculations were repeated with the quantity-quality ratio in place of N, P, K, and Na. Because QF = N + P + K, and QN, QP, and QK are percentages of QF, there would be a perfect correlation with QF, so one of the qualities (QP) should be left out for a solution. A subset of 20 variety levels (one variety had to be dropped because of program limitations) and a subset of eight replication levels were also used in the above models. The R^2 values are also given without the variety and replication correction for comparison (Table 6). These models for gross sugar and tons/acre are very significant and also show that the variability from replications and varieties is highly significant. In these models, few single terms were significant, but the variability accounted for ranged from 47% to 53%.

The models for sugar percentage and impurity indices were also very highly significant. All terms within the model were significant,

Table 5.—Stepwise regression models with dependent Y_k and independent X_i = 1 to 12 and 13 to 24.

Variables in model (no.)	Total dry matter		Gross sugar		Tons/acre		Sugar Percentage		Impurity Index	
	R^2	Y_1	R^2	Y_2	R^2	Y_3	R^2	Y_4	R^2	Y_5
Dry Pulp Measurements										
1	0.3587	Na	0.1106	Ca	0.1222	P	0.4391	Na	0.4630	K
2	0.4410	K	0.1532	P	0.1570†	Ca	0.4949	K	0.7073	N
3	0.4593†	B	0.1935	Mg	0.1744	K	0.5337	B	0.7517	Na
4	0.4623	S	0.2137†	Zn	0.1900	Mg	0.5488†	Ca	0.7645	Mn
5	0.4662*	Ca	0.2246	B	0.1997	B	0.5548	P	0.7699	Fe
6	0.4671	Mn	0.2389	Fe	0.2090	Fe	0.5599	Mg	0.7757	Mg
7	0.4692	N	0.2518	Na	0.2205	Zn	0.5634	Zn	0.7793	Cu
8	0.4698	Fe	0.2587*	Mn	0.2299*	Mn	0.5662*	Cu	0.7831†	P
9	0.4703	Cu	0.2590	S	0.2310	S	0.5668	S	0.7850	S
10	0.4709	Zn	0.2596	K	0.2312	Na	0.5681	N	0.7869*	Ca
11	0.4709	Mg	0.2600	Cu	0.2312	Cu	0.5693	Fe	0.7873	B
12	0.4709	P	0.2600	N	0.2312	N	0.5696	Mn	0.7873	Zn
Leaf Blade Measurements										
1	0.0625	B	0.3076	P	0.3382	P	0.0533	K	0.0685	K
2	0.0979	Ca	0.3579	Zn	0.4111	Zn	0.0752	B	0.1292	B
3	0.1080	Cu	0.3781	Ca	0.4662	Ca	0.0893	Zn	0.1521	Mn
4	0.1162	K	0.4027	Na	0.4783	Na	0.0984	Fe	0.1804	Zn
5	0.1256	Zn	0.4171†	S	0.4968†	B	0.1072*	Mn	0.1960	Na
6	0.1332*	Na	0.4237	B	0.5049	Mg	0.1117	Cu	0.2153†	Mg
7	0.1361	Fe	0.4289*	Mg	0.5107*	S	0.1169	P	0.2249	Cu
8	0.1382	Mn	0.4311	K	0.5133	Fe	0.1213	Ca	0.2308	N
9	0.1390	S	0.4316	Fe	0.5163	Cu	0.1226	Na	0.2370*	Fe
10	0.1398	Mg	0.4321	Mn	0.5183	Mn	0.1230	S	0.2374	P
11	0.1399	N	0.4324	Cu	0.5188	K	0.1232	M	0.2375	S
12	0.1400	P	0.4326	N	0.5195	N	0.1234	Mg	0.2375	Ca

* Most efficient Model

† Model with all terms significant

Table 6.—Regression models with dependent Y_k and $Y_i = X_1$ to X_4 and correction for variety and replication levels (dry-pulp measurements).

Variable	df	Mean square	Calc. F	Variable	df	Mean square	Calc. F
		Y_2 = Gross sugar				With quantity-quality factors Y_2 = Gross sugar	
Total	159			Total	159		
1 = N	1	28.95×10^3	NS	1 = QF	1	11.82×10^5	NS
2 = P	1	10.48×10^5	NS	2 = QN	1	18.32×10^5	NS
3 = K	1	90.47×10^4	NS	3 = QK	1	55.11×10^4	NS
4 = Na	1	43.82×10^5	4.93*	4 = Var.	19	16.14×10^5	1.79*
5 = Var.	19	16.38×10^5	1.84*	5 = Rep.	7	68.34×10^5	7.58**
6 = Rep.	7	66.81×10^5	7.52**	Model	29	36.51×10^5	4.05**
Model	30	36.79×10^5	4.14*	Error	130	90.18×10^4	
Error	129	88.77×10^4		R^2 =		0.4746	
R^2 =		0.4946					
$R^2\dagger$ =		0.1313		$R^2\dagger$ =		0.1047	
		Y_3 Tons/acre				Y_3 = Tons/acre	
Total	159			Total	159		
1 = N	1	4.98	NS	1 = QF	1	52.64	5.84*
2 = P	1	1.96	NS	2 = QN	1	2.11	NS
3 = K	1	51.82	5.77*	3 = QK	1	1.65	NS
4 = Na	1	13.39	NS	4 = Var.	19	24.14	2.68**
5 = Var.	19	24.57	2.73**	5 = Rep.	7	71.03	7.88**
6 = Rep.	7	72.30	8.05**	Model	29	44.81	4.97**
Model	30	43.80	4.88**	Error	130	9.02	
Error	129	8.98		R^2 =		0.5257	
R^2 =		0.5315					
$R^2\dagger$ =		0.1366		$R^2\dagger$ =		0.1366	
		Y_4 = Sucrose				Y_4 = Sucrose	
Total	159			Total	159		
1 = N	1	1.0174	4.21*	1 = QF	1	4.5259	16.73**
2 = P	1	1.9943	8.25**	2 = QN	1	4.4493	16.45**
3 = K	1	6.0106	24.87**	3 = QK	1	6.2366	23.06**
4 = Na	1	5.2998	21.93**	4 = Var.	19	2.1832	8.07**
5 = Var.	19	1.3285	5.50**	5 = Rep.	7	1.2740	4.71**
6 = Rep.	7	0.6659	2.75*	Model	29	3.1600	11.68**
Model	30	31.8880	131.93**	Error	130	0.2705	
Error	129	0.2417		R^2 =		0.7226	
R^2 =		0.7546					
$R^2\dagger$ =		0.5106		$R^2\dagger$ =		0.3133	
		Y_5 = Sucrose				Y_5 = Index	
Total	159			Total	159		
1 = N	1	60.05×10^3	32.44**	1 = QF	1	20.15×10^4	103.55**
2 = P	1	59.40×10^2	NS	2 = QN	1	16.85×10^3	8.65**
3 = K	1	12.50×10^4	67.53**	3 = QK	1	39.55×10^3	20.32**
4 = Na	1	15.76×10^3	8.52**	4 = Var.	19	41.43×10^2	2.13**
5 = Var.	19	23.34×10^2	NS	5 = Rep.	7	56.11×10^2	2.88**

Table 6—Continued

Variable	df	Mean square	Calc. F	Variable	df	Mean square	Calc. F
6 = Rep.	7	37.31×10^2	NS	Model	29	41.44×10^2	2.13**
Model	30	36.00×10^2	19.45**	Error	130	19.46×10^2	
Error	129	18.50×10^2		R^2 =		0.8076	
R^2 =		0.8185					
R^2† =		0.7593		R^2†, =		0.7134	

* = Significant at the 5% level.
** = Significant at the 1% level.
† = Without replication or variety correction

except for Y_5, where the P term and variety and replication subjects were not significant. This non-significance must be because Y_5 was a function of N, K, Na, and sugar percentage and was not affected by variety and replication variation.

Curvilinear Models

The study of the three-dimensional surfaces indicated there were some interactions and at least squared relationships involved (Fig. 1). Because all surfaces were similar, a second degree polynomial with all linear interactions was calculated. This model included only N, P, K, and Na in the dry pulp, with N^2, P^2, K^2, and Na^2 and interactions $N \times P$, $N \times K$, $N \times Na$, $P \times K$, $P \times Na$, and $K \times Na$, a total of 14 terms (Table 7). This model had too many terms, because the R^2 value dropped only 1% with seven, or one half, of the total terms. The most efficient models accounted for 52%, 56% and 78% of the variability, with nine, seven, and seven terms on Y_1, Y_4, and Y_5, respectively. The same models as above were calculated on leaf-blade measurements, confirming the results and accounting for more variability with Y_2 and Y_3 than in the linear models.

For comparison, a model was calculated by the stepwise method with the four X_i: i = X_1 to X_4 measurements, X^2i and X^3i, a total of 12 with Y_k, k = 3 to 5 only (Table 8). For Y_3, the most efficient models had seven variables, with R^2 of 0.18. For Y_4, the most efficient model had eight variables, with R^2 of 0.57. For Y_5, the most efficient model had eight variables, with R^2 of 0.78.

Discussion

The simple correlations were erratic and not as strong as expected. Perhaps the time of sampling the plot was not at the best for the fertility level of the field used. Also, the effects of the varieties could have confounded the results, as observed when correction was made for varieties and replication levels in the regression models. Variaties and replications were significant in most models. The three-dimensional graphs were not complete enough, as evidenced by the number of missing cells in the graphs. The blade and petiole measurements, although tending

Table 7.—Second-degree polynomial curvilinear models with N, P, K, and Na measured in dry pulp.

Variables in model (No.)	Total dry matter		Gross Sugar		Tons/acre		Sugar percentage		Impurity Index	
	R^2	Stepwise Variable order	R^2	Stepwise Variable order	R^2	Stepwise Variable order	R^2	Stepwise Variable order	R^2	Stepwise Variable order
1	0.3587	Na	0.0344	N × P	0.1000	N × P	0.4391	Na	0.4630	K
2	0.4409	K	0.1044	N	0.1289	N^2	0.4953	K^2	0.7133	N^2
3	0.4489	N × K	0.1293†	K × Na	0.1370	N^2	0.5003	N	0.7336	P × Na
4	0.4819	N^2	0.1422	N × K	0.1396	Na^2	0.5262	N × K	0.7689†	N × P
5	0.5037†	N × Na	0.1600	P × K	0.1540	Na	0.5435	N × P	0.7719	P
6	0.0539	P	0.1688	N^2	0.1671	K^2	0.5569†	N × Na	0.7768	p^2
7	0.5129	P^2	0.1778*	P × Na	0.1761†	P × K	0.5631*	P × K	0.7791	N × Na
8	0.5194	K × Na	0.1816	Na^2	0.1805	N × Na	0.5655	N^2	0.7816	Na
9	0.5226*	P × Na	0.1841	Na	0.1823	P × Na	0.5675	K	0.7840*	Na^2
10	0.5247	N × P	0.1844	P	0.1845	K × Na	0.5690	K × Na	0.7848	K^2
11	0.5260	Na^2	0.1847	K	0.1863	K	0.5693	P	0.7858	P × K
12	0.5264	K^2	0.1855	p^2	0.1873*	P	0.5695	P × Na	0.7862	N × K
13	0.5267	P × K	0.1857	N × Na	0.1887	P^2	0.5696	p^2	0.7864	K × Na
14	0.5267	N	0.1858	K^2	0.1887	N × K	0.5696	Na^2	0.7865	N

* Most efficient model

† All terms significant in model.

Table 8.—Third-degree polynomial curvilinear models, with N, P, K, and Na measured in dry pulp.

Variables in model (no.)	Y_3 = Tons/acre R^2	Stepwise variable order	Y_4 = Sugar Percentage R^2	Stepwise variable order	Y_5 = Impurity Index R^2	Stepwise variable order
1	0.12	P	0.44	Na	0.46	K
2	0.14	K^2	0.50	K^2	0.71	N^2
3	0.14	N^3	0.50	N^2	0.75	Na
4	0.14	N^2	0.50	N^3	0.77	P^3
5	0.15	Na^3	0.53	N	0.77	N
6	0.17	Na^2	0.54	Na^2	0.77	P^2
7	0.18	N*	0.57	Na^3	0.78	P*
8	0.18	P^2	0.57	P*	0.78	N^3
9	0.19	P^3	0.57	P^2	0.78	K^3
10	0.19	K^3	0.57	P^3	0.78	K^2
11	0.19	K	0.58	K^3	0.78	Na^3
12	0.19	Na	0.58	K	0.78	Na^2

* Most efficient model

to confirm the dry-pulp surfaces, were not very pronounced, partly because of the low fertility gradient in the test. N was low toward the end of the season.

The linear models with measurements from all three plant materials accounted for the most variability and were the most uniform among the Y_k's. We thought that Mn, Cu, Zn, Fe, and B should be kept together as a group if included in a model, because we know that they all must be present at all times. The leaf-blade measurements alone added significantly to models with Y_2 and Y_3, but not to models with Y_1, Y_4, and Y_5 (Table 4).

The second-degree polynomials, with only the common variables $X_i = X_1$ to X_4, accounted for only 6% more variability over that of the linear model with $X_i = X_1$ to X_4. These models had too many terms, because the R^2 value with 14 df for Y_4 was 0.57 and dropped to only 0.56 with 7 df for the most efficient model (Table 1). The R^2 value of the corresponding model with only $X_i = X_1$ to X_4 in the linear form was 0.51.

Hecker, et al (12) reported that the cubic model was probably the best form for determining a threshold value in thin juice purity measurements. Our comparison gave evidence that the cubic form included too many terms in the model for efficiency because the complete 12-term model accounted for 58% of the variability, whereas the most efficient model with 7 terms accounted for 57% of the variability (Table 8).

Summary

Linear models with more than one plant material measurement were more consistent over all the Y_k dependent variables. These models were most efficient with 12 to 16 variables.

The inclusion of the micronutrients, with the four X_1 = N, P, K, Na, did not account for enough added variability to justify their use.

The second- and third-degree polynomials had excess terms when all variables of the polynomial were included. They were not substantially better than comparable linear models. Some justification must be found for the terms that are included. Three-dimensional surfaces should help in this division if the coefficients of variation are high enough.

For calculating a general regression model, variety test data could be used if it covered wide enough fertility levels.

Literature Cited

(1) BALDWIN, C. S. and C. K. STEVENSON. 1969. The effect of nitrogen on yield percent sucrose and clear juice purity of sugarbeets. J. Am. Soc. Sugar Beet Technol. 15: 522-527.

(2) BARTON, C. J. 1948. Photometric analysis of phosphate rock. Anal. Chem. 20: 1068-1073.

(3) BROWN, R. J. and R. R. WOOD. 1952. Improvement of processing quality of sugar beets by breeding methods. Proc. Am. Soc. Sugar Beet Technol. 7:314-318.

(4) DAHLBERG, H. W. 1950. Chemical methods for breeding sugar beets. Proc. Am. Soc. Sugar Beet Technol. 6: 137-138.

(5) DOXTATOR, C. W. and H. M. BAUSERMAN. 1952. Parent-progeny tests for sodium and potassium content. Proc. Am. Soc. Sugar Beet Technol. 7:319-321.

(6) DRAPER, N. R. and H. SMITH. 1966. Applied regression analysis John Wiley and Sons, Inc., New York.

(7) FINKER, R. E. and H. M. BAUSERMAN. 1956. Breeding of sugar beets with reference to sodium, sucrose and raffinose content. J. Am. Soc. Sugar Beet Technol. 9: 170-177.

(8) FINKNER, R. E., D. B. OGDEN, P. C. HANZAS, and R. F. OLSON. 1958. II. The effect of fertilizer treatment on the calcium, sodium, potassium, raffinose, galactinol, nine amino acids and total amino acid content of three varieties of sugar beet grown in the Red River Valley, Minnesota. J. Am. Soc. Sugar Beet Technol. 10: 272-280.

(9) GERRITZ, H. W. 1935. Methods of digesting biological materials for calcium and phosphorus analysis. Ind. and Eng. Chem. Anal. Ed. 7: 167-168.

(10) HADDOCK, JAY L., GEORGE K. RYSER, and J. C. THEURER, 1968. Mineral composition of sugarbeet plants as affected by varieties and genotypes. J. Am. Soc. Sugar Beet Technol. 15: 454-461.

(11) HADDOCK, JAY L. and DARREL M. STUART. 1964. Critical levels versus quantity-quality factors for assessing nutritional status of sugarbeet plants. J. Am. Soc. Sugar Beet Technol. 13: 42-58.

(12) HECKER, RICHARD J. W. T. FEDERER, LE ROY POWERS, and MERLE G. PAYNE. 1967. Relationships of four yield characters with five chemical characters in sugarbeets. J. Am. Soc. Sugar Beet Technol. 14: 555-573.

(13) Hermann, H. and R. Ratner. 1962. The influence of sodium and potassium-sodium ratio on the sugar content of beets and on their processing. The International Sugar Beet Journal 64: 136-138.

(14) Jackson, M. L. 1958. Soil chemical analysis. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

(15) Muller, K., A. Numann, and W. Werner. 1963. The influence of nitrogen-potassium ratio on yield and quality of sugar beet. Zucker 15: 1-6.

(16) Rorabaugh, Guy and Lloyd W. Norman. 1956. The effect of various impurities on crystallization of sucrose. J. Am. Soc. Sugar Beet Technol. 9: 238-252.

(17) Ryser, George K. 1965. A regression study on tare samples of sugar beets in relation to factors influencing productivity and quality. J. Am. Soc. Sugar Beet Technol. 13: 727-747.

(18) Ryser, George K., Myron Stout, Albert Ulrich, and F. V. Owen. 1959. Some chemical and physiological characteristics of inbred lines of sugar beets. J. Am. Soc. Sugar Beet Technol. 10: 525-543.

(19) Ryser, George K. and J. C. Theurer. 1970. Impurity index selections on individual sugarbeets. J. Am. Soc. Sugar Beet Technol. 16: 399-407.

(20) Tolman, Bion and Ronald C. Johnson, 1958. Effect of nitrogen on yield and sucrose content of sugar beets. J. Am. Soc. Sugar Beet Technol. 10: 254-257.